Quiz 5 Solutions– Wednesday, July 28

Let $f(x) = 2x^3 - 3x^2 - 12x + 1$, defined on $(-\infty, \infty)$.

1. (2 points) Find the critical points of f.

   $f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$.

   To find the critical points, we let $f'(x) = 0$, so we obtain that $x = 2$ and $x = -1$ are the critical points of f.

2. (2 points) Find the intervals on which f increases and decreases.

   We have to study the sign of f' on three intervals:

   - On $(-\infty, -1)$, f' is positive, so f is increasing.
   - On $(-1, 2)$, f' is negative, so f is decreaing.
   - On $(2, \infty)$, f' is positive, so f is increasing.

3. (2 points) Find the x values where the local minimum and maximum values of f occur.

   Using the First Derivative Test, we can see that -1 is a local maximum, and 2 is a local minimum.
   Alternatively, we can use the Second Derivative Test, for the critical points of f.
   We have that $f''(x) = 6(2x - 1)$. Now, $f''(-1) < 0$, so -1 is a local maximum, and $f''(2) > 0$, so 2 is a local minimum.

4. (3 points) Find the intervals on which f is concave down and concave up. Find the inflection points of f.

We let $f''(x) = 6(2x - 1) = 0$, and we get that $x = \frac{1}{2}$ is a point where the sign of f'' might change. We get in fact that:

- On $(-\infty, \frac{1}{2})$, f'' is negative, so f is concave down.
- On $(\frac{1}{2}, \infty)$, f'' is positive, so f is concave up.
- $x = \frac{1}{2}$ is an inflection point for f.

5. (1 point) Does f have an absolute maximum or minimum? Justify.

$\lim_{x \to \infty} f(x) = \infty$ so f does not have an absolute maximum.
$\lim_{x \to -\infty} f(x) = -\infty$ so f does not have an absolute minimum.