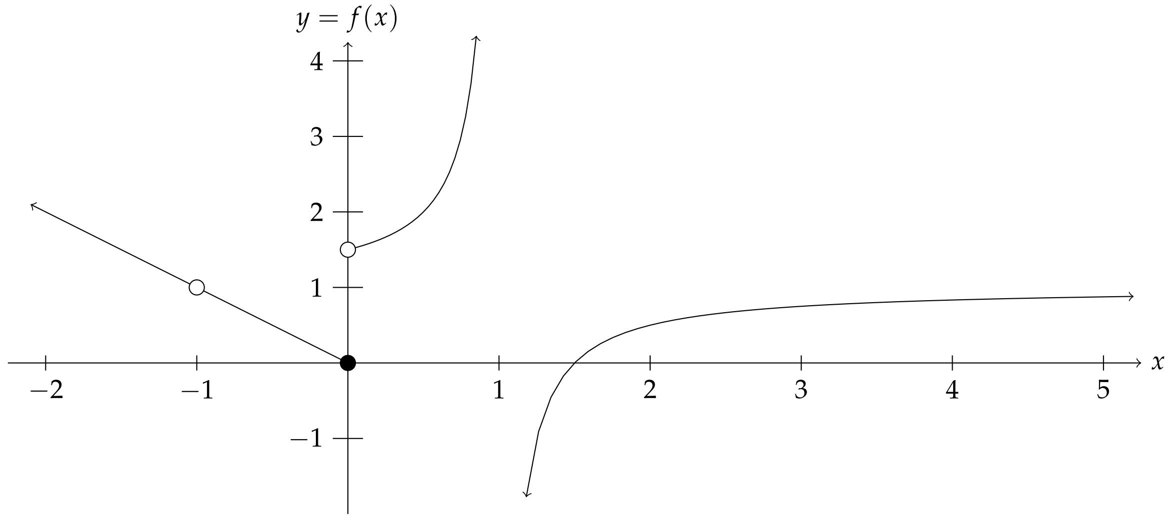
Quiz 2 — Wednesday, July 7

Name: Solution Key

1. (*4 points*) Let $f(x) = \begin{cases} \dfrac{-x^2 - x}{x+1}, & x \le 0 \\ \dfrac{2x-3}{2(x-1)}, & x > 0 \end{cases}$. A graph of f over $[-2, 5]$ is given below.



Describe the types and locations of the discontinuities of f and the asymptotes of f. You do not need to justify your answers.

- f has a removable discontinuity at $x = -1$
- f has an infinite discontinuity at $x = 1$
- f has a jump discontinuity at $x = 0$
- The graph has vertical asymptote(s) at $x = 1$.
- The graph has horizontal asymptote(s) at $y = 1$.

2. (*2 points*) Evaluate $\lim\limits_{x\to 3} \dfrac{\frac{1}{3}-\frac{1}{x}}{x-3}$, if it exists.

Solution: Since substituting in $x = 3$ gives the indeterminate expression $\frac{0}{0}$, we must instead rewrite the function:

$$\lim_{x\to 3} \frac{\frac{1}{3}-\frac{1}{x}}{x-3} = \lim_{x\to 3} \frac{\frac{x-3}{3x}}{x-3} = \lim_{x\to 3} \frac{1}{3x} = \boxed{\frac{1}{9}}$$

3. (*2 points*) Evaluate $\lim\limits_{x\to\infty} \sqrt{9x^2+x} - 3x$, if it exists.

Solution: Taking the limit of each term as $x \to \infty$ yields $\infty - \infty$, so we must rewrite the function. We use the conjugate radical:

$$\begin{aligned}
\lim_{x\to\infty} \sqrt{9x^2+x} - 3x &= \lim_{x\to\infty} (\sqrt{9x^2+x} - 3x) \cdot \frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x} \\
&= \lim_{x\to\infty} \frac{(9x^2+x)-9x^2}{\sqrt{9x^2+x}+3x} \\
&= \lim_{x\to\infty} \frac{x}{\sqrt{9x^2+x}+3x} \cdot \frac{1/x}{1/x} \\
&= \lim_{x\to\infty} \frac{1}{\sqrt{9+\frac{1}{x}}+3} = \boxed{\frac{1}{6}}
\end{aligned}$$

4. (*2 points*) Use the Intermediate Value Theorem to show that the equation $2^x = 2 - x$ has a solution c in the interval $(0,1)$.

Solution: Let $f(x) = 2^x + x - 2$, so that $f(c) = 0$ exactly when c is a solution to the equation. We also note f is continuous at all real x. Since

$$f(0) = 1 + 0 - 2 = -1 < 0 \quad \text{and} \quad f(1) = 2 + 1 - 2 = 1 > 0,$$

the Intermediate Value theorem states that there is some value c in $(0,1)$ so that $f(c) = 0$. Thus, the equation has a solution c in $(0,1)$.