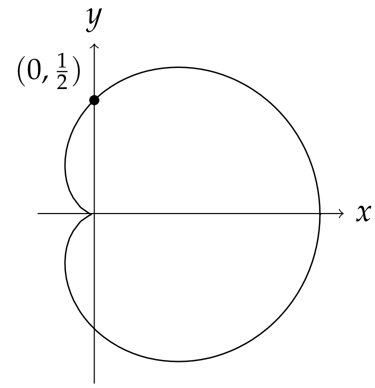
Problems: Extrema, Implicit Differentiation, Related Rates

1. A curve called a *cardioid* is described by the equation

$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2.$$

Find the tangent line to the curve at the point $(0, \frac{1}{2})$.



cardioid

2. A plane flying at an altitude of 3 miles will pass directly over a radar station. The radar station measures that, at time T, the distance between the plane and the station is 5 miles, and the plane is approaching the station at 200 miles per hour. What is the speed of the plane relative to the ground?

3. Let $f(x) = \dfrac{3x+4}{x^2+1}$, defined on $(-\infty, \infty)$.

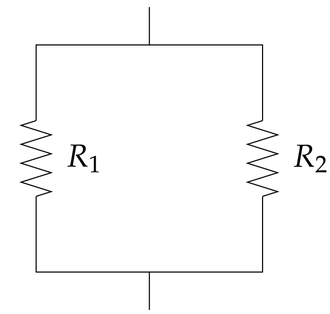
(a) Find the critical numbers of $f(x)$.

(b) Which critical numbers correspond to local maxima? Local minima? Justify your answer using the First or Second Derivative Tests.

(c) What are the absolute maximum and minimum values of f, if they exist? Explain.

4. If two resistors with resistances R_1 and R_2 are wired in parallel, the total equivalent resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$



Suppose that at time T, $R_1 = 30\ \Omega$ and is increasing at $3\,\Omega/\text{s}$, and that $R_2 = 60\,\Omega$ and is decreasing at $3\,\Omega/\text{s}$.

(a) How fast does R change with respect to time at time T?

(b) What rate of change of R_2 would make $\dfrac{dR}{dt} = 0$?

5. A trough is 12 feet long and has a cross-section shaped like an isosceles triangle, with width 5 feet at the top and height 3 feet. If the trough is filled with water at a rate of 10 ft^3/min, how fast is the water level rising when the water is 2 feet deep?

6. Let $f(x) = x^{\sqrt{x}}$, defined for $x > 0$.

(a) Compute $f'(x)$.

(b) Find the critical numbers of $f(x)$. Which ones correspond to local minima? Local maxima?

(c) Find the absolute maximum and minimum of $f(x)$ on the interval $\left[\frac{1}{16}, 4\right]$.