

Practice Problems on Limits and Continuity

1 A tank contains 10 liters of pure water. Salt water containing 20 grams of salt per liter is pumped into the tank at 2 liters per minute.

1. Express the salt concentration $C(t)$ after t minutes (in g/L).
2. What is the long-term concentration of salt, i.e., $\lim_{t \rightarrow \infty} C(t)$?

2 Find the values of a and b that make $f(x)$ continuous for all real x .

$$f(x) = \begin{cases} be^x + a + 1, & x \leq 0 \\ ax^2 + b(x + 3), & 0 < x \leq 1 \\ a \cos(\pi x) + 7bx, & x > 1 \end{cases}$$

3 Sketch the graph of a function f with the following properties:

- $\lim_{x \rightarrow 1} f(x) = 2$, but $f(1) = 1$
- $\lim_{x \rightarrow 3} f(x) = +\infty$
- $\lim_{x \rightarrow 2^+} f(x) = -1$, $\lim_{x \rightarrow 2^-} f(x) = 3$
- $\lim_{x \rightarrow +\infty} f(x) = -2$
- $\lim_{x \rightarrow -\infty} f(x) = -\infty$

4 Show that the equation $\sqrt{x-5} = \frac{1}{x+3}$ has at least one real solution.

5 Consider the rational function

$$f(x) = \frac{x^5 - x^4 - 2x^3}{x^4 - 3x^3 - x^2 + 3x}$$

- For what values of a does f have a removable discontinuity at a ? What is $\lim_{x \rightarrow a} f(x)$ at those a ?
- For what values of a does f have an infinite discontinuity at a ?
- What is $\lim_{x \rightarrow +\infty} f(x)$?

(Hint: Factor the numerator and the denominator.)

6 Find the value of a such that

$$\lim_{x \rightarrow -1} \frac{2x^2 - ax - 14}{x^2 - 2x - 3}$$

exists. What is the value of the limit?