Midterm #2 — April 12, 2013, 10:00 to 10:53 AM

Name: ________________________________

Circle your recitation:

R01 (Claudio · Fri) R02 (Xuan · Wed) R03 (Claudio · Mon)

- **You have a maximum of 53 minutes.** This is a closed-book, closed-notes exam. No calculators or other electronic aids are allowed.
- Read each question carefully. Show your work and justify your answers for full credit. You do not need to simplify your answers unless instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

Grading

1	/30
2	/15
3	/20
4	/15
5	/20
Total	/100

1. (*30 points*) Find the general solution to each of the following differential equations:

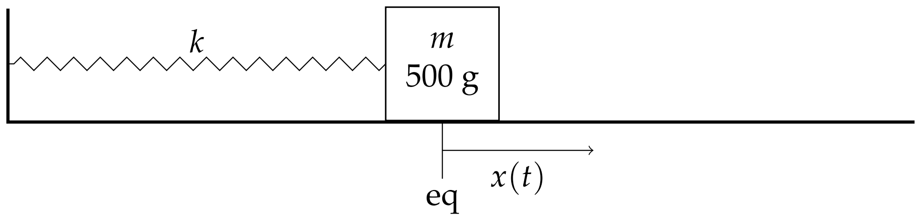
(a) (*10 points*) $y'' - 4y' + 3y = 0$

(b) (*10 points*) $y'' - 4y' + 4y = 0$

(c) (*10 points*) $y'' - 4y' + 5y = 0$

2. (*15 points*) Two solutions to the DE $y'' - 3y' - 10y = 0$ are $y_1 = e^{5x}$ and $y_2 = e^{-2x}$. Find a solution to this DE satisfying the initial conditions $y(0) = 7$ and $y'(0) = 7$.

3. (*20 points*) A 500-gram test mass m is attached to a spring of unknown spring constant k and allowed to settle into its equilibrium position, as shown:



The mass is struck sharply at time $t = 0$, and the resulting displacement measured to be

$$x(t) = 0.25e^{-3t} \sin 5t \qquad (x \text{ in meters, } t \text{ in seconds})$$

(a) (*5 points*) Is this system underdamped, critically damped, or overdamped? Explain.

(b) (*10 points*) Find the spring constant k of the spring and the damping constant c resulting from natural friction in the system. Include appropriate units.

(c) (*5 points*) Suppose we then apply a periodic force $f(t) = 10\cos\omega t$ (in newtons) to the spring-mass system, where we may vary the forcing frequency ω. Find the value of ω at which the system exhibits practical resonance, or explain why it does not occur.

4. (*15 points*) Find a particular solution to the nonhomogeneous DE

$$y^{(3)} - 6y'' + 12y' = 40e^{2x} - 24.$$

5. (*20 points*) Let $y_1(x) = x$ and $y_2(x) = x^4$.

(a) (*10 points*) Show that y_1 and y_2 are both solutions to the DE $x^2y'' - 4xy' + 4y = 0$.

(b) (*5 points*) Show that y_1 and y_2 are linearly independent functions on the entire real line.

(c) (*5 points*) Find the general solution to the nonhomogeneous DE

$$x^2y'' - 4xy' + 4y = 6x^4 - 9x.$$