Midterm #1 — March 6, 2013, 10:00 to 10:53 AM

Name: _____

Circle your recitation:

R01 (Claudio · Fri) R02 (Xuan · Wed) R03 (Claudio · Mon)

- **You have a maximum of 53 minutes.** This is a closed-book, closed-notes exam. No calculators or other electronic aids are allowed.
- Read each question carefully. Show your work and justify your answers for full credit. You do not need to simplify your answers unless instructed to do so.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do not unstaple or detach pages from this exam.

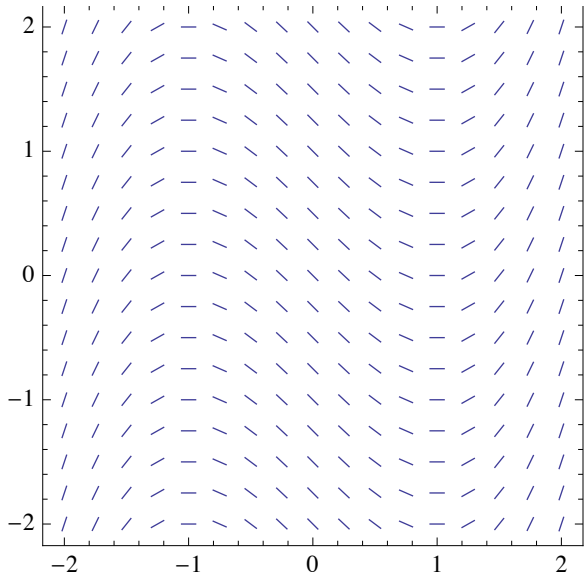
Grading

1	/10
2	/15
3	/15
4	/15
5	/15
6	/15
7	/15
Total	/100

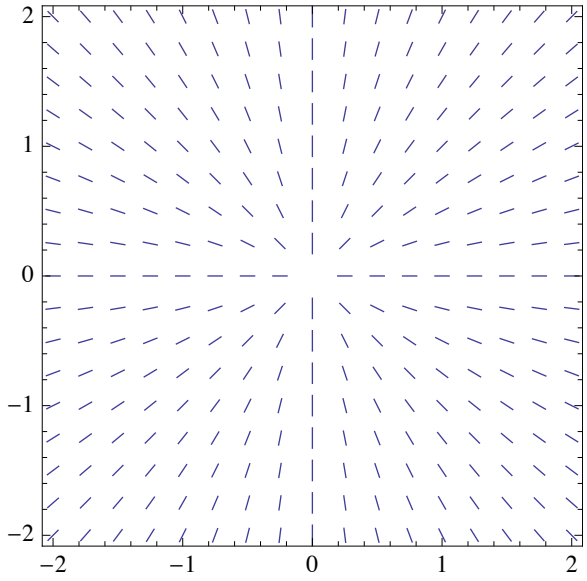
1. (*10 points*)
For each of the slope fields below, identify the differential equation A–E matching it:

A. $y' = y^2 - 1$ B. $xy' = y$ C. $y' = xy$ D. $y' = x^2 - 1$ E. $y' = y - x$

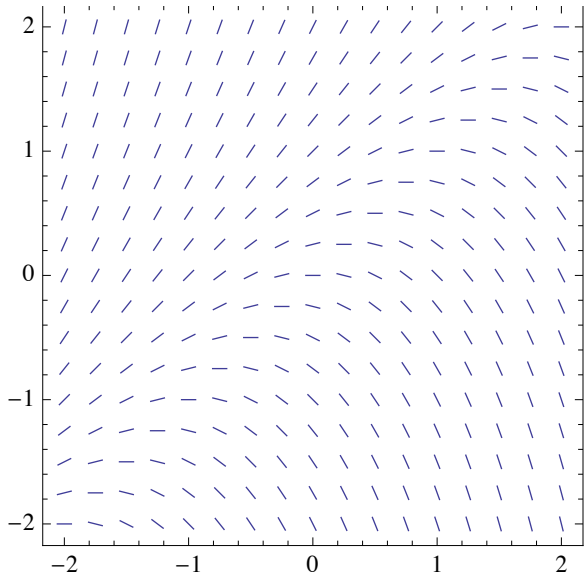
I. _____



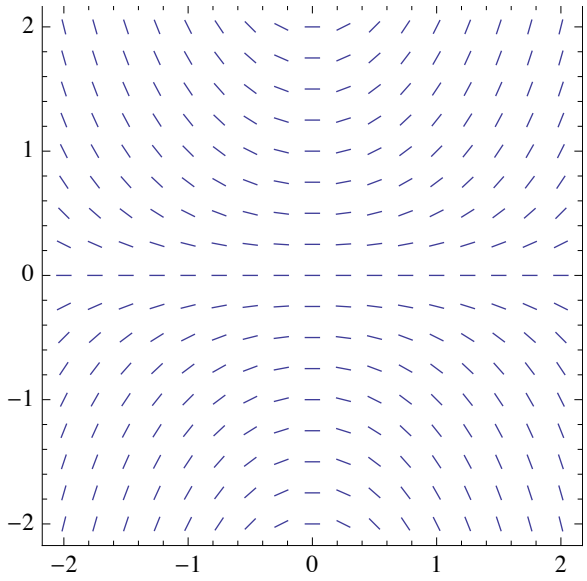
II. _____



III. _____



IV. _____



2. (*15 points*) Solve the initial value problem $xy' = 4y + 4x^6$, $y(1) = 1$.

3. (*15 points*) Find the general solution to the differential equation $2xyy' + y^2 + x^2 = 0$.

4. (*15 points*) A cup of hot coffee at 190°F is placed outside, where it is currently 30°F. After 10 minutes, the coffee has cooled to 150°F.

(a) (*5 points*) Write a differential equation governing the temperature T of the coffee as a function of the time t since it is placed outside. Explain any parameters you include.

(b) (*5 points*) Find $T(t)$.

(c) (*5 points*) What is the temperature of the coffee after another 10 minutes?

5. (*15 points*) Find the solution $y(x)$ to the initial value problem $yy'' = (y')^2 - 6y'$ with $y(0) = 1, y'(0) = 8$.

6. (*15 points*) Consider the differential equation $y' = 4y^2 - y^3 - 4y$.

(a) (*6 points*) Find the equilibria of this differential equation.

(b) (*3 points*) Construct a phase diagram for this differential equation.

(c) (*3 points*) Characterize the stability of the equilibria you found in part (a).

(d) (*3 points*) Describe the expected behavior of a solution $y(t)$ with initial value $y(0) = 4$ as t increases.

7. (*15 points*) Consider the following two differential equations:

(I) $(x + 2y)\,dx + y\,dy = 0$ **(II)** $(x + 2y)\,dx + (2x - 3y)\,dy = 0$

(a) (*7 points*) Which of the equations is exact? Explain.

(b) (*8 points*) Find the general solution to one of the equations.

(c) (*Extra credit: 5 points*) Find the general solution to the other equation.