# Midterm #1 Practice Problems

**1.** Solve the following initial value problems:

(a) $xy' = y + x^2$, $y(1) = 0$

(b) $y' = 6e^{2x-y}$, $y(0) = 0$

(c) $y' = -\frac{2}{x}y + \frac{1}{x^2}$, $y(1) = 2$

(d) $(1+x)y' = 4y$, $y(0) = 1$

(e) $v' - \frac{1}{x}v = xv^6$, $v(1) = 1$

(f) $2xyy' = x^2 + 2y^2$, $y(1) = 2$

**2.** Find the (complete) general solution to each of the following differential equations:

(a) $y' + 2xy + 6x = 0$

(b) $(3x^2 + 2y^2)\,dx + (4xy + 6y^2)\,dy = 0$

(c) $xy'' + y' = 12x^2$

(d) $9y' = xy^2 + 5xy - 14x$

(e) $y' + \frac{2}{3x}y + 3y^{-2} = 0$

(f) $y' + y\cot x = \cos x$

(g) $x^2y' + \frac{1}{x}y^3 = 2y^2$

(h) $(xy + y^2)\,dx + x^2\,dy = 0$

(i) $y' = (\frac{4}{x} + 1)y$

(j) $(2x - \frac{\ln y}{x^2})\,dx + \frac{1}{xy}\,dy = 0$

**3.** Consider the differential equation $\dfrac{dy}{dx} = \dfrac{4y}{x^2-4}$.

(a) Find all values $a$ and $b$ such that this equation with the initial condition $y(a) = b$ is guaranteed to have a unique solution.

(b) Find the general solution to the differential equation.

(c) Sketch a slope field for this equation, along with several representative solutions. Are there any points where solutions exist but are not unique?

**4.** A cold drink at 36°F is placed in a sweltering conference room at 90°F. After 15 minutes, its temperature is 54°F.

(a) Find the temperature $T(t)$ of the drink after $t$ minutes, assuming it obeys Newton's law of cooling.

(b) How long does it take for the drink to reach 74°F?

**5.** A tank with capacity 500 liters originally contains 200 l of water with 100 kg of salt in solution. Water containing 1 kg salt per liter is entering at a rate of 3 l/min, and the mixture is allowed to flow out of the tank at a rate of 2 l/min.

(a) Find the amount of salt in the tank at any time $t$ before it starts to overflow.

(b) Find the concentration (in kg/l) of salt in the tank at the time when the tank overflows.

(c) If the tank has infinite volume, what is the limiting concentration in the tank?

**6.** Suppose that at time $t = 0$, half of a "logistic" population of 100,000 persons have heard a certain rumor, and that the number of those who have heard it is then increasing at the rate of 1000 persons per day. How long will it take for this rumor to spread to 80% of the population?

**7.** Consider the differential equation $y' + \tan y = 0$.

(a) Find all equilibrium solutions to this equation, and characterize their stability, with justification.

(b) Find the general solution to the differential equation, and sketch a few representative solutions.

**8.** Consider the differential equation $y' = y^2 - 4y - 4k^2 + 8k$ with parameter $k$.

(a) Given $k = 2$, find the critical points of the differential equation, draw a phase diagram, and determine the stability of the critical points.

(b) Draw the bifurcation diagram for this differential equation, and label the stability of the equilibria.

**9.** A boat moving at 27 m/s suddenly loses power and starts to coast. The water resistance slows the boat down with force proportional to the 4/3-power of its velocity. Suppose the boat takes 20 seconds to slow down to 8 m/s. What is the total distance the boat travels as it slows down to a stop?