Homework #11 Solutions

Problems

Bolded problems are worth 2 points.

- Section 5.5: 6, **12**
- Section 7.1: **2**, 12, **14**, 20, **32**, **46**, 64
- Section 7.3: 2, **6**, 12, **16**, **34**

5.5.6. The marginal cost function for a company is given by

$$C'(q) = q^2 - 16q + 70 \text{ dollars/unit},$$

where q is the quantity produced. If $C(0) = 500$, find the total cost of producing 20 units. What is the fixed cost and what is the total variable cost for this quantity?

Solution: The fixed cost is the cost at $q = 0$. From the problem statement, this is 500 dollars. The total variable cost is the integral of the marginal cost from $q = 0$ to $q = 20$:

$$\int_0^{20} C'(q)\, dq = \int_0^{20} q^2 - 16q + 70\, dq.$$

An antiderivative for $q^2 - 16q + 70$ is $F(q) = \frac{1}{3}q^3 - 8q^2 + 70q$, so this integral is

$$\int_0^{20} C'(q)\, dq = F(20) - F(0) = \left(\frac{1}{3}(20)^3 - 8(20)^2 + 70(20)\right) - 0 \approx 866.66\dots.$$

Therefore, the total cost of 20 units is $866.66 + 500 = 1366.66$ dollars. ■

5.5.12. The net worth, $f(t)$, of a company is growing at a rate of $f'(t) = 2000 - 12t^2$ dollars per year, where t is in years since 2005. How is the net worth of the company expected to change between 2005 and 2015? If the company with worth \$40,000 in 2005, what is it worth in 2015?

Solution: The change in the net worth of a company is given by the definite integral

$$\int_0^{10} f'(t)\, dt = \int_0^{10} 2000 - 12t^2\, dt.$$

To evaluate this, we find an antiderivative $G(t)$ for $2000 - 12t^2$. By the antiderivative rules, or by inspection, $G(t) = 2000t - 4t^3$. Then this integral is

$$\int_0^{10} 2000 - 12t^2\, dt = G(10) - G(0) = (2000(10) - 4(10)^3) - 0 = 16{,}000,$$

so the company's net worth increased by 16,000 from 2005 to 2015. Its total net worth in 2015 is $16{,}000 + 40{,}000 = 56{,}000$ dollars. ■

7.1.2. Find an antiderivative of $f(t) = 5t$.

Solution: One antiderivative is given by $F(t) = 5\left(\frac{1}{2}t^2\right) = \frac{5}{2}t^2$. Any other antiderivative will be of the form $\frac{5}{2}t^2 + C$ for some constant C. ■

7.1.12. Find an antiderivative of $f(x) = x + x^5 + x^{-5}$.

Solution: Applying the power rule for antiderivatives with $n = 1$, $n = 5$, and $n = -5$, we have

$$F(x) = \frac{1}{2}x + \frac{1}{6}x^6 + \frac{1}{-4}x^{-4} = \frac{1}{2}x + \frac{1}{6}x^6 - \frac{1}{4}x^{-4}$$

as an antiderivative of $f(x)$. ■

7.1.14. Find an antiderivative of $g(z) = \sqrt{z}$.

Solution: Since $g(z) = \sqrt{z} = z^{1/2}$, we use the power rule for antiderivatives with $n = 1/2$. Then $n + 1 = 3/2$, so $\frac{1}{n+1} = \frac{2}{3}$. One antiderivative is then

$$G(z) = \frac{2}{3}z^{3/2}.$$

■

7.1.20. Find an antiderivative of $q(y) = y^4 + \frac{1}{y}$.

Solution: Using the power rule and the log rule for antiderivatives, we find that an antiderivative for $q(y)$ is

$$Q(y) = \frac{1}{5}y^5 + \ln|y|.$$

■

7.1.32. Find an antiderivative $F(x)$ for $f(x) = e^x$ with $F(0) = 0$. Is there only one possible solution?

Solution: The general antiderivative for $f(x) = e^x$ is $F(x) = e^x + C$. We set $x = 0$: then $F(0) = 0$, but $F(0) = e^0 + C = 1 + C$. Therefore, $1 + C = 0$, so $C = -1$. Hence, the specific antiderivative is

$$F(x) = e^x - 1,$$

and we can easily see that its derivative is $f(x) = e^x$. Since $C = -1$ is the only value of C such that the general antiderivative $F(x) = e^x + C$ satisfies $F(0) = 0$, this is the only solution. ■

7.1.46. Find the indefinite integral $\int e^{2t}\,dt$.

Solution: Using the exponential rule for antiderivatives with $k = 2$, this indefinite integral is

$$\int e^{2t}\,dt = \frac{1}{2}e^{2t} + C.$$

■

7.1.64. Find an antiderivative $F(x)$ with $F'(x) = x^2 + 1$ and $F(0) = 5$.

Solution: Since $F'(x) = x^2 + 1$, $F(x) = \frac{1}{3}x^3 + x + C$ for some C. We also require that $F(0) = 5$, so $\frac{1}{3}0^3 + 0 + C = 5$. Therefore, $C = 5$, so $F(x) = \frac{1}{3}x^3 + x + 5$ is the solution. ■

7.3.2. Evaluate the definite integral $\int_0^4 6x\,dx$ exactly.

Solution: We find an antiderivative $F(x)$ for $6x$. Using the power rule, we take $F(x) = 3x^2$. Then

$$\int_0^4 6x\,dx = F(4) - F(0) = 3(4)^2 - 3(0)^2 = 48.$$

■

7.3.6. Evaluate the definite integral $\int_1^4 \frac{1}{\sqrt{x}}\,dx$ exactly.

Solution: Let $f(x) = \dfrac{1}{\sqrt{x}}$ be the integrand. Then $f(x) = \frac{1}{x^{1/2}} = x^{-1/2}$, so to find its antiderivative we use the power rule with $n = -1/2$. Therefore, $n + 1 = 1/2$, so $\frac{1}{n+1} = 2$, and an antiderivative is

$$F(x) = 2x^{1/2} = 2\sqrt{x}$$

Therefore, the integral is

$$\int_1^4 \frac{1}{\sqrt{x}}\,dx = F(4) - F(1) = 2\sqrt{4} - 2\sqrt{1} = 2(2) - 2 = 4 - 2 = 2.$$

■

7.3.12. Evaluate the definite integral $\int_0^1 y^2 + y^4\,dy$ exactly.

Solution: Using the power rule, an antiderivative for $y^2 + y^4$ is

$$F(y) = \frac{1}{3}y^3 + \frac{1}{5}y^5.$$

Therefore,

$$\int_0^1 y^2 + y^4\,dy = F(1) - F(0) = \left(\frac{1}{3} + \frac{1}{5}\right) - (0 + 0) = \frac{8}{15}.$$

■

7.3.16. Evaluate the definite integral $\int_0^1 2e^x\,dx$ exactly.

Solution: An antiderivative for the integrand $2e^x$ is $F(x) = 2e^x$, so

$$\int_0^1 2e^x\,dx = F(1) - F(0) = 2e^1 - 2e^0 = 2e - 2.$$

■

7.3.34. Oil is leaking out of a ruptured tanker at the rate of $r(t) = 50e^{-0.02t}$ thousand liters per minute.

(a) At what rate, in liters per minute, is oil leaking out at $t = 0$? At $t = 60$?

(b) How many liters leak out during the first hour?

Solution (a): At $t = 0$, $r(0) = 50e^{-0.02(0)} = 50e^0 = 50$ liters per minute. At $t = 60$, $r(60) = 50e^{-0.02(60)} = 50e^{-1.2} \approx 15.06$ liters per minute. ■

Solution (b): The total volume lost in the first 60 minutes is $\int_0^{60} r(t)\,dt$, so we find an antiderivative $G(t)$ for $r(t)$:

$$G(t) = 50 \cdot \frac{1}{-0.02}e^{-0.02t} = -2500e^{-0.02t}.$$

Therefore,

$$\int_0^{60} r(t)\,dt = G(60) - G(0) = -2500e^{-1.2} + 2500 = 2500\left(1 - e^{-1.2}\right),$$

in liters. Numerically, this value is approximately 1747 liters. ■