Homework #5 Solutions

Problems

Bolded problems are worth 2 points.

- Section 2.3: 10, **16**, 26

- Section 2.4: **2**, 6, **10**, 22, **28**

- Section 3.1: 4, **14**, 24, **28**, 36, **38**, 50, **60**

2.3.10. On May 9, 2007, CBS Evening News had a 4.3 point rating. (Ratings estimate the number of viewers.) News executives estimated that a 0.1 drop in the ratings for the CBS Evening News corresponds to a \$5.5 million drop in revenue. Express this information as a derivative. Specify the function, the variables, the units, and the point at which the derivative is evaluated.

Solution: Let a denote the rating, in points, and R the revenue, in millions of dollars. Since the revenue depends on the rating, R is a function $R(a)$ of a. The observed rate of change is

$$\frac{\Delta R}{\Delta a} = \frac{-5.5}{-0.1} = 55 \text{ million \$/point.}$$

This rate of change happens when the rating is 4.3, so $R'(4.3) = 55$ million \$/point. ∎

2.3.16. The cost, C (in dollars), to produce g gallons of a chemical can be expressed as $C = f(g)$. Using units, explain the meaning of the following statements in terms of the chemical:
(a) $f(200) = 1300$ (b) $f'(200) = 6$

Solution (a): If $f(200) = 1300$, it costs \$1300 to make 200 gallons of this chemical. ∎

Solution (b): If $f'(200) = 6$, the cost increases at \$6 per gallon once 200 gallons of the chemical have already been made. ∎

2.3.26. Suppose that $f(x)$ is a function with $f(20) = 345$ and $f'(20) = 6$. Estimate $f(22)$.

Solution: Since $f'(20) = 6$ is the rate of change of f at $x = 20$, f should increase by about $(6)(2) = 12$ as x changes from 20 to 22. Hence, we estimate the total value of $f(22)$ to be $345 + 12 = 357$. ∎

2.4.2. At one of the labeled points on the graph in Figure 2.39 both dy/dx and d^2y/dx^2 are positive. Which is it?

Solution: We look for where both the first derivative dy/dx and the second derivative d^2y/dx^2 are positive, so where y is increasing and is concave up. This combination occurs only at B. ∎

2.4.6. Give the sign of the first and second derivative of the function graphed below. Each derivative is either positive everywhere, zero everywhere, or negative everywhere.

Solution: The function shown in the graph is decreasing, so its first derivative is negative. It is also concave up, so its second derivative is positive. ∎

2.4.10. Use the following values of t and $w(t)$ to decide the following questions:

t	100	110	120	130	140
$w(t)$	10.7	6.3	4.2	3.5	3.3

(a) Does the derivative of $w(t)$ appear to be positive or negative over the given interval? Explain.

(b) Does the second derivative of $w(t)$ appear to be positive or negative over the given interval? Explain.

Solution (a): We estimate $w'(t)$ over each of these four 10-unit intervals:

t	100	110	120	130	140
$w'(t)$	-0.42	-0.21	-0.07	-0.02	

Since they are all negative, we expect $w'(t)$ to be as well. ∎

Solution (b): Since the $w'(t)$ values are becoming less negative as t increases, they are increasing, so $w''(t)$ is positive. ∎

2.4.22. Sketch the graph of a function f such that $f(2) = 5$, $f'(2) = 1/2$, and $f''(2) > 0$.

Solution: One such graph is as follows (the tangent line at $x = 2$ is also drawn in gray):

∎
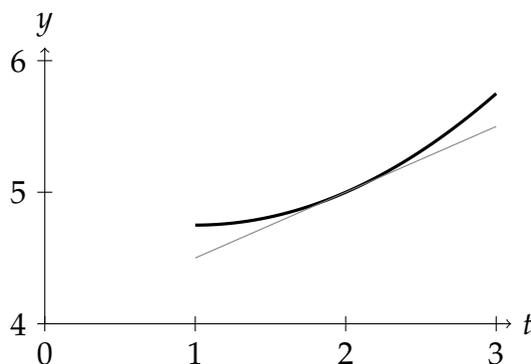
2.4.28. Each of the graphs in the problem shows the position of a particle moving along the x-axis as a function of time, $0 \le t \le 5$. The vertical scales of the graphs are the same. During this time interval, which particle has:

(a) Constant velocity?

(b) The greatest initial velocity?

(c) The greatest average velocity?

(d) Zero average velocity?

(e) Zero acceleration?

(f) Positive acceleration throughout?

Solution (a): If the particle has constant velocity, the graph will have constant slope, so will be a straight line. This graph is IV. ∎

Solution (b): The particle with the greatest initial velocity will have the graph with the steepest upward slope at time 0. This graph is III. ∎

Solution (c): For the average velocity, we consider the slope of the secant line between the left and right endpoints of the graph. The secant line of graph II has the steepest slope. ∎

Solution (d): The secant line of graph I has 0 slope, so this particle has zero average velocity. ∎

Solution (e): The acceleration corresponds to the second derivative of position, and hence to the concavity of the graphs. Of the graphs, only IV has no concavity and thus zero acceleration. ∎

Solution (f): Positive acceleration corresponds to a concave up graph, and only graph II is concave up on the entire interval $[0, 5]$. ∎

3.1.4. Find the derivative of $y = x^{-12}$.

Solution: The derivative is $y' = -12x^{-13}$. ∎

3.1.14. Find the derivative of $y = 8t^3 - 4t^2 + 12t - 3$.

Solution: The derivative is $y = 8(3t^2) - 4(2t) + 12 - 0 = 24t^2 - 8t + 12$. ∎

3.1.24. Find the derivative of $y = z^2 + \dfrac{1}{2z}$.

Solution: Writing $y = z^2 + \frac{1}{2}z^{-1}$, we have $y = 2z + \frac{1}{2}(-z^{-2}) = 2z - \dfrac{1}{2z^2}$. ■

3.1.28. Find the derivative of $h(\theta) = \theta(\theta^{-1/2} - \theta^{-2})$.

Solution: We write $h(\theta) = \theta^{1/2} - \theta^{-1}$. Then $h'(\theta) = \frac{1}{2}\theta^{-1/2} - (-1)\theta^{-2} = \frac{1}{2}\theta^{-1/2} + \theta^{-2}$. ■

3.1.36. Find the derivative of $h(x) = \dfrac{ax + b}{c}$, where a, b, c are constant.

Solution: Write $h(x) = \frac{ax}{c} + \frac{b}{c} = \frac{a}{c}x + \frac{b}{c}$. Then $h'(x) = \frac{a}{c} + 0 = \frac{a}{c}$. ■

3.1.38. Let $f(x) = x^3 - 4x^2 + 7x - 11$. Find $f'(0), f'(2), f'(-1)$.

Solution: First, we have $f'(x) = 3x^2 - 8x + 7$. We then evaluate this function:

$$f'(0) = 0 - 0 + 7 = 7$$
$$f'(2) = 3(4) - 8(2) + 7 = 12 - 16 + 7 = 3$$
$$f'(-1) = 3(1) - 8(-1) + 7 = 3 + 8 + 7 = 18$$
■

3.1.50.

(a) Find the equation of the tangent line to $f(x) = x^3$ at the point where $x = 2$.

(b) Graph the tangent line and the function on the same axes. If the tangent line is used to estiamte values of the function, will the estimates be overestimates or underestimates?

Solution (a): First, we find the point where the tangent line touches the graph: the x coordinate is 2, and the y coordinate is 8. Next, the slope of the tangent line is $f'(2)$, so we compute that: $f'(x) = 3x^2$, so $f'(2) = 3(4) = 12$. Hence, from the point slope formula $y = m(x - a) + b$ for a line of slope m through the point (a, b), the line is

$$y = 12(x - 2) + 8 = 12x - 24 + 8 = 12x - 16.$$
■

Solution (b): Here is a graph of $y = x^3$, together with the tangent line (in gray):

$$y = x^3$$
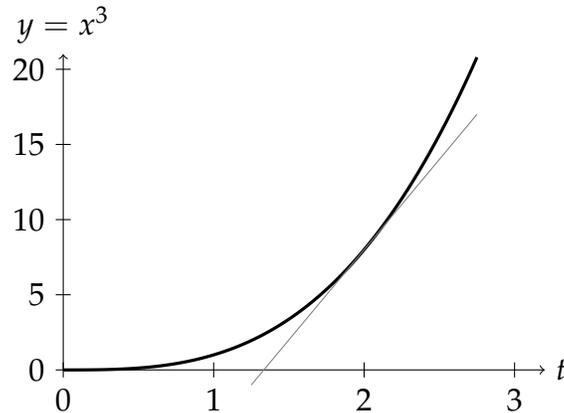
The tangent line is below the graph itself, so if its values are used as estimates for the function, they will be underestimates. ∎

3.1.60. A ball is dropped from the top of the Empire State Building. The height, y, of the ball above the ground (in feet) is given as a function of time, t (in seconds), by

$$y = 1250 - 16t^2.$$

(a) Find the velocity of the ball at time t. What is the sign of the velocity? Why is this to be expected?

(b) When does the ball hit the ground, and how fast is it going at that time? Give your answer in feet per second and in miles per hour (1 ft/sec = 15/22 mph).

Solution (a): The velocity of the ball is given by the derivative $y' = -32t$. This velocity is negative, because the ball is falling downward. ∎

Solution (b): The ball hits the ground when the height y is 0, so when $1250 - 16t^2 = 0$. Then $t^2 = 1250/16$, so $t = \sqrt{\frac{1250}{16}} \approx 8.84$ s. The velocity is then

$$v = -32t \approx -32(8.84) \approx -282.9 \, \frac{\text{ft}}{\text{sec}}$$

In miles per hour, we have

$$v \approx -282.9 \, \frac{\text{ft}}{\text{sec}} = -282.9 \left(\frac{15}{22} \frac{\text{ft}}{\text{sec}} \right) \approx -192.9 \text{ mph}.$$

∎