**Score**

Section I:_____

Section II:_____

Total off: _____

Percent: _____

# Stony Brook

**STATE UNIVERSITY OF NEW YORK**

**MAT 122 Midterm 2 – Spring '09**

Last Name: KEY First Name: _________ Recitation: R____

**Part I:** Show all work in the space provided and write your final answer in the answer box. [5 points each for questions 1 - 7]

**Directions:** For the following questions, find the derivative using the formulas developed in class. The only time you have to use the definition is in question 6. Leave the answer in simplest form with positive exponents. Factor the answer where possible.

1. $\frac{d}{dx}\left(6\sqrt[3]{x}-\frac{1}{x^3}\right) = \frac{d}{dx}\left(6x^{1/3}-x^{-3}\right)$

$\boxed{2x^{-2/3}+\frac{3}{x^4}}$

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2. $\frac{d}{dx}\left(x^2e^x\right)$ (Leave the answer in factored form.)

$= \frac{d}{dx}(x^2)e^x + x^2\frac{d}{dx}(e^x)$

$= 2xe^x + x^2e^x$

$= x(x+2)e^x$

$\boxed{x(x+2)e^x}$

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3. Find $\frac{d}{dx}\left(\frac{2e^x}{\ln x}\right) = 2\left(\frac{e^x\ln x - e^x\frac{1}{x}}{(\ln x)^2}\right) = \boxed{\frac{2e^x(x\ln x-1)}{x(\ln x)^2}}$

4. Find $f'(1)$ if $f(x) = \ln x^{\frac{2}{3}}$ $f(x) = \frac{2}{3} \ln x$ by log rules

$$f'(x) = \frac{2}{3} \cdot \frac{1}{x} = \frac{2}{3x}$$

$$f'(1) = \frac{2}{3(1)}$$

$$\boxed{f'(1) = \frac{2}{3}}$$

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5. If $y = 2x^3 - 3x^2 + e^x$ find $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = 6x^2 - 6x + e^x$$

$$\frac{d^2y}{dx^2} = 12x - 6 + e^x$$

$$\boxed{12x - 6 + e^x}$$

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**Directions:** For the following question, find the derivative by the method specified.

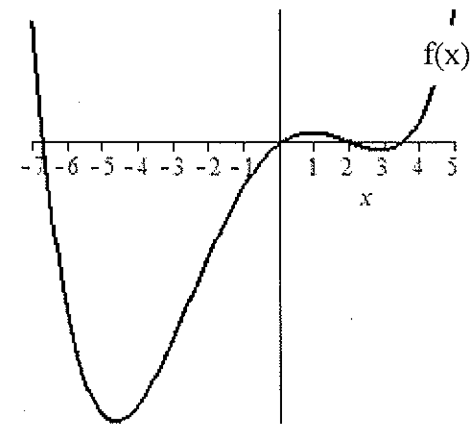
6. Use the **definition** of the derivative to find $f'(x)$ if $f(x) = x^2 + 2x + 1$. Feel free to check using the "short-cut" formulas.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) + 1 - (x^2+2x+1)}{h}$$

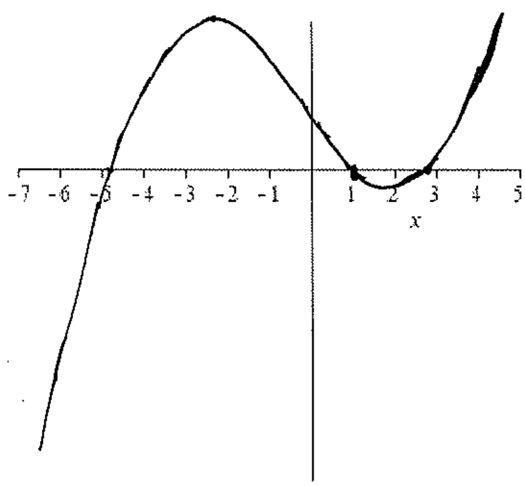
$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h \cancel{+1} - \cancel{x^2} - \cancel{2x} \cancel{-1}}{h}$$

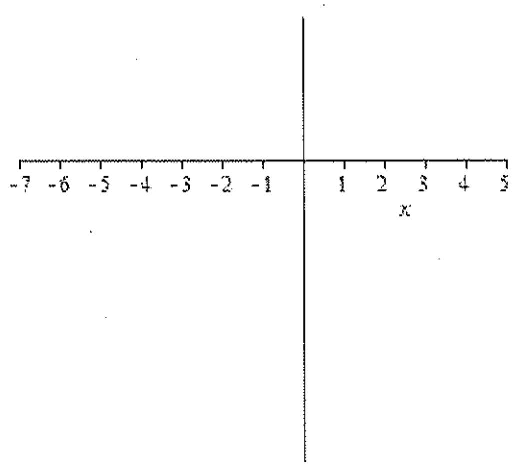
$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}}(2x + h + 2) = \boxed{2x + 2}$$

7. Below is a graph of a function f(x). Use the grid provided to sketch the graph of the derivative, f '(x). An extra grid is provided. If you used more than one grid circle the graph you want marked.




Extra grid if needed



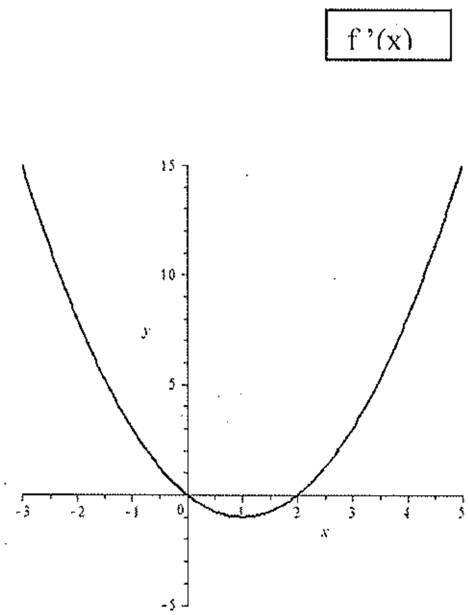



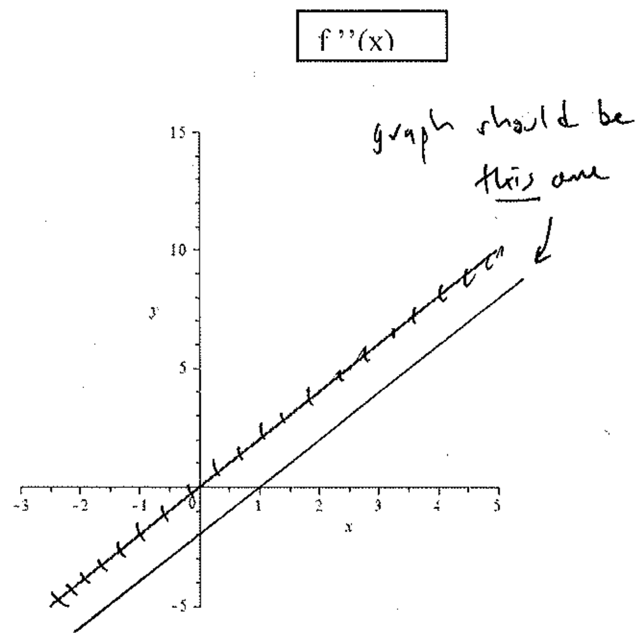

**Part II:** [10 points each]

8. The graphs of the first and second **derivatives** of function f(x) (not shown) appear below.

   a) Complete the table by filling in the word from the left side of the table for the given value of x that describes the graph of the original function f(x) at that point.

Entries are based on graphs below




| x = | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|
| **Increasing, Decreasing or Constant** | I | I | I | C | D | C | I | I | I |
| **Concave up, Concave down or Linear** | D | D | D | D | L | U | U | U | U |

Remember, these describe f(x), the <u>original</u> function which is <u>not</u> shown

b) In the answer box indicate which of the following graphs is best described by the properties in the table above. (Two more appear on the next page.)

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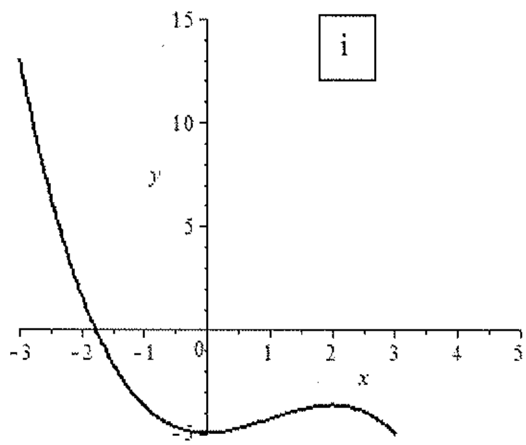


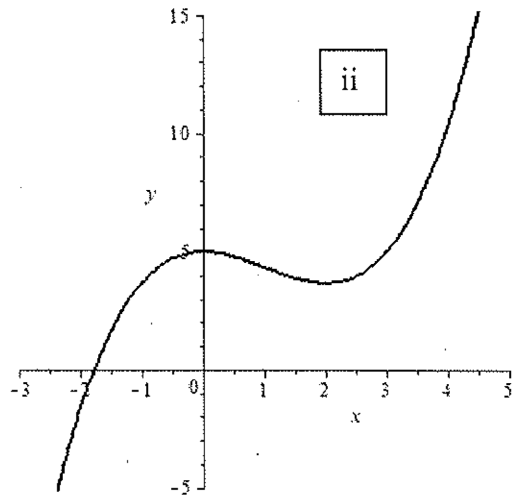


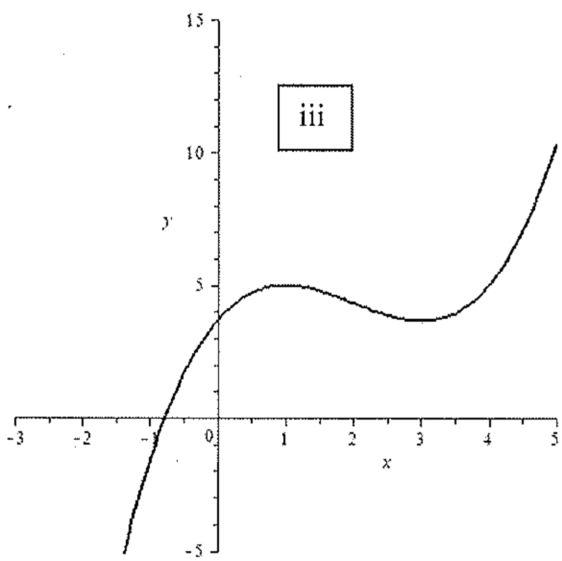


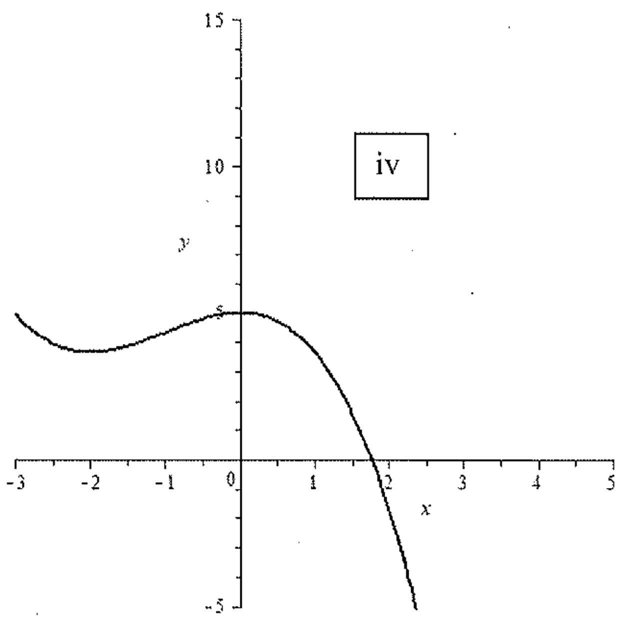

9 . Show all work in the space provided. Circle your final answer:

a) Find $f'(x)$ if $f(x) = x + x^{-2} + 1$

$$f'(x) = 1 - 2x^{-3}$$

b) Write an equation of the line tangent to the curve $f(x) = x + x^{-2} + 1$ at x = 1.
Show all work

$$f'(1) = 1 - 2(1)^{-3} = 1 - 2 = -1$$

$$f(1) = 1 + 1^{-2} + 1 = 3$$

line:
$$y - 3 = -1(x-1) = 1 - x$$
$$y = 4 - x$$

c) Check by graphing $f(x) = x + x^{-2} + 1$ and your answer to b) on the same axes using the given window on the grid below. An extra set is available in case you need it. Graph carefully, accuracy counts!

```
WINDOW
 Xmin=-4.7
 Xmax=4.7
 Xscl=1
 Ymin=-3.1
 Ymax=5.1
 Yscl=1
 Xres=1
```

Extra Axes