Part I : ______
#6: ______
#7: ______
#8: ______
Total off: ______
Percent: ______

Stony Brook

STATE UNIVERSITY OF NEW YORK

MAT 122 Final Exam S'11

Last Name: Key **First Name :** Sol ution

I.D. # ______________ **Recitation: R**____

Part I: Show <u>all work</u> in the space provided and write your final answer in the answer box. [5 points each]

Directions: For the following questions, find the derivative or integral <u>using the formulas</u> developed in class. Leave the answer in simplest form with positive exponents. Factor the answer where possible.

1. $\frac{d}{dx}(\ln(x^2+x)) = \frac{2x+1}{x^2+x}$

2.

$\frac{d}{dx}\left(e^{3x^2}\right) = 6x\,e^{3x^2}$

3. Find $\int\left(2e^x - \frac{2}{x}\right)dx = 2e^x - 2\ln|x| + c$

4. Find $\int\left(12x^3 - 3\sqrt{x} + 5\right)dx$ $= 3x^4 - 2x^{3/2} + 5x + C$

$\boxed{}$

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5. Find $\int_1^8 \left(2x^{\frac{1}{3}}\right)dx$ $= \frac{3}{2}x^{4/3}\Big|_1^8 = \frac{3}{2}(16-1) = \frac{3}{2}\cdot 15 = \frac{45}{2}$

$\boxed{}$

Part II Show <u>all work</u> in the space provided and circle your final answer

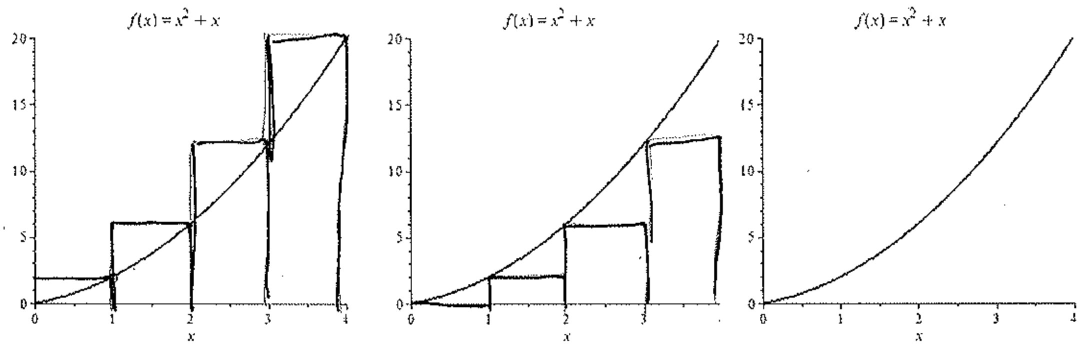
6. On the next page is a graph of the function $f(x) = x^2 + x$. (There is an extra copy in case you need it). [10 points]

   a) Estimate the area under the curve from x = 0 to x = 4 using the **Right-Hand-Sum** method with 4 sub-intervals and using the **Left-Hand-Sum** method with 4 sub-intervals. Draw the rectangles on the given graphs and show the calculations below. Remember these are only an approximation. You will be asked to find the exact area in part b). There is a spare grid in case you make a mistake. If you use it, be sure to label the graph as either **LHS**, or **RHS**

[**Continued on the next page**]

6. (Continued)

RHS **LHS** **Extra Grid**__________



a) Show calculations here:

RHS:

$2 + 6 + 12 + 20 = 40$

LHS:

$0 + 2 + 6 + 12 = 20$

b) Find the exact area by evaluating an appropriate integral. This must be done algebraically and work shown. Feel free to check using your calculator.

$$\text{area} = \int_0^4 x^2 + x \, dx = \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 \right]_0^4$$

$$= \left(\frac{64}{3} + 8 \right) - (0 + 0) = \frac{88}{3} \approx 29.333\ldots$$

7. Answer the questions below for the function $f(x) = x^3 - 3x^2 - 9x + 11$ on the interval All parts of this question must be done algebraically except d)

a) Find $f'(x)$ and $f''(x)$.

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

b) Find the *critical points* of f, and give your answers as ordered pairs (x, y). Classify the points as *relative minimum, relative maximum* or neither in the table below. Use as many rows as you need,

Point	Classification
(3, -16)	loc min
(-1, 16)	loc max

solve $f'(x) = 0$: $3x^2 - 6x - 9 = 0$

$3(x-3)(x+1) = 0$

$\rightarrow x = 3, \; x = -1$ are critical points

Second Derivative test:

$f''(3) = 18 - 6 = 12$; $f''(-1) = -6 - 6 = -12$

c) Find any *inflection points* of f. Give your answers as ordered pairs (x, y).

$f''(x) = 0 \rightarrow 6x - 6 = 0, \Rightarrow x = 1$

$f(1) = 0$ IP: $(1, 0)$

d) Check by graphing f(x) on the given grid using the given window. An extra grid is provided in case you need it.

```
WINDOW
 Xmin=-4.7
 Xmax=4.7
 Xscl=1
 Ymin=-20
 Ymax=22
 Yscl=5
 Xres=■
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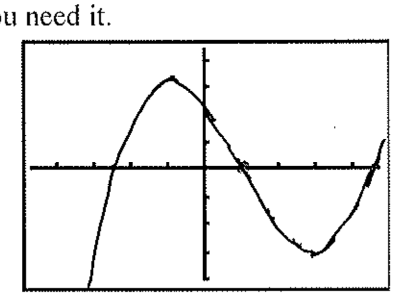
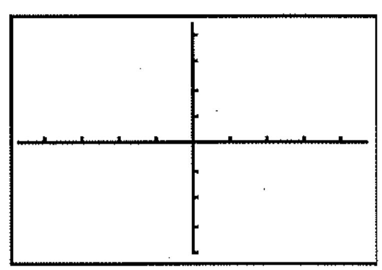




8. The *rate of change*, ***F'(x)***, of the function ***F(x)*** is given by the graph below [10 points]

a) Find the *total change* over the following intervals:

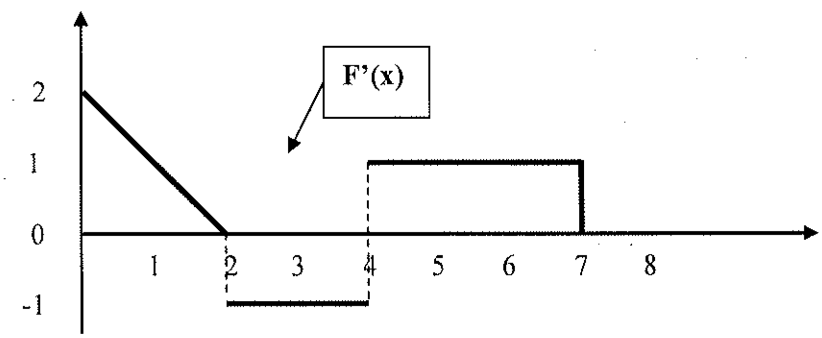
i) [0, 2] ______ +2

ii) [2, 4] ______ −2

iii) [2, 7] ______ +1

iv) [0, 7] ______ +3

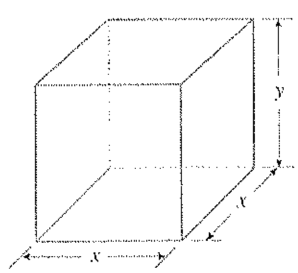
b) If $F(0)=1$ find $F(7)$ ______ 4



Bonus Question

This problem is optional but if you solve it successfully you will get 5 extra points added to the grade on this exam. It must be solved algebraically using calculus and all work must be shown. Use the back of this sheet.

MINIMIZING PACKAGING COSTS A rectangular box is to have a square base and a volume of 20 ft^3. If the material for the base costs 30¢/square foot, the material for the sides costs 10¢/square foot, and the material for the top costs 20¢/square foot, determine the dimensions of the box that can be constructed at minimum cost.



Bonus Problem:

Height y : volume $= x^2 y = 20$, $\underline{y = \frac{20}{x^2}}$

Cost of box: sides + top + bottom

$$c(x) = 4xy \cdot (0.10) + 0.20\,x^2 + 0.30\,x^2$$
$$= 0.40\,x \cdot \frac{20}{x^2} + 0.50\,x^2$$
$$= \frac{8}{x} + \frac{1}{2}x^2 \qquad \text{(in dollars)}$$

$$c'(x) = -\frac{8}{x^2} + x = 0$$

$$\Rightarrow x^3 = 8, \quad \underline{x = 2}$$

(can check is a global minimum)

$$\text{height} = \frac{20}{2^2} = \frac{20}{4} = 5$$

– box is 2 ft square at base,
5 ft high.