Homework #2 Solutions

Problems

- Section 1.3: 8, 10, 16, 30, 38, 50
- Section 1.5: 2, 10, 18, 22, 24, 28

1.3.8. Find the relative, or percent, change in W if it changes from 0.3 to 0.05.

Solution: The percent change is

$$R = \frac{\Delta W}{W} = \frac{0.05 - 0.3}{0.3} = \frac{-0.25}{0.3} = -\frac{5}{6}.$$

In a decimal expansion, this is $0.8333\ldots$, or $83.333\ldots\%$. ■

1.3.10. For which pairs of consecutive points in Figure 1.33 (see book) is the function graphed:

(a) Increasing and concave up

(b) Increasing and concave down

(c) Decreasing and concave up

(d) Decreasing and concave down

Solution:

(a) Intervals (D, E) and (H, I) are increasing and concave up.

(b) Intervals (A, B) and (E, F) are increasing and concave down.

(c) Intervals (C, D) and (G, H) are decreasing and concave up.

(d) Intervals (B, C) and (F, G) are decreasing and concave down. ■

1.3.16. Table 1.12 gives the net sales of The Gap, Inc., in millions of dollars.

Year	2003	2004	2005	2006	2007	2008
Sales	15,854	16,267	16,019	15,923	15,763	14,526

(a) Find the change in net sales between 2005 and 2008.

(b) Find the average rate of change in net sales between 2005 and 2008. Give units and interpret your answer.

(c) From 2003 to 2008, were there any one-year intervals during which the rate of change was positive? If so, when?

Solution (a): The change in net sales is $14{,}526 - 16{,}019 = -1493$. ■

Solution (b): The average rate of change is

$$\frac{-1493}{3} = -497.67,$$

in units of millions of dollars per year. This means that sales are decreasing. ■

Solution (c): Yes, from 2003 to 2004 the sales increased by 413 million dollars, giving a positive rate of change. ■

1.3.30. The number of US households with cable television was 12,168,450 in 1977 and 73,365,880 in 2003. Estimate the average rate of change in the number of US households with cable television during this 26-year period. Give units and interpret your answer.

Solution: The average rate of change is

$$\frac{73{,}365{,}880 - 12{,}168{,}450}{2003 - 1977} = \frac{61{,}197{,}430}{26} = 2{,}353{,}747.3,$$

in units of households per year. Thus, on average, 2.35 million households added cable per year. ■

1.3.38. Values of $F(t)$, $G(t)$, and $H(t)$ are in Table 1.21. Which graph is concave up and which is concave down? Which function is linear?

t	10	20	30	40	50	60
$F(t)$	15	22	28	33	37	40
$G(t)$	15	18	21	24	27	30
$H(t)$	15	17	20	24	29	35

Solution: We compute the average rates of change for these functions in the intervals between these t-values:

t	10 to 20	20 to 30	30 to 40	40 to 50	50 to 60
$F(t)$	0.7	0.6	0.5	0.4	0.3
$G(t)$	0.3	0.3	0.3	0.3	0.3
$H(t)$	0.2	0.3	0.4	0.5	0.6

Since the rates of change for F are decreasing, it is concave down. Since the rates of change for G are constant, it is linear. Since the rates of change for H are increasing, it is concave up. ■

1.3.50. During 2008 the US economy stopped growing and began to shrink. Table 1.23 gives quarterly data on the US Gross Domestic Product (GDP) which measures the size of the economy.

t (years since 2008)	0	0.25	0.5	0.75	1
GDP (trillion \$)	14.15	14.29	14.41	14.2	14.09

(a) Estimate the relative growth rate (percent per year) at the first four times in the table.

(b) Economists often say an economy is in recession if the GDP decreases for two quarters in a row. Was the US in recession in 2008?

Solution (a): The percent rates of change per quarter:

t (years since 2008)	0	0.25	0.5	0.75
relative change in GDP	0.98%	0.84%	−1.46%	−0.77%

■

Solution (b): Yes, the US was in recession during the last two quarters of 2008, since the growth rate was negative for those two consecutive quarters. ■

1.5.2. Each of the following functions gives the amount of a substance present at time t. In each case, give the amount present initially (at $t = 0$), state whether the function represents exponential growth or decay, and give the percent growth or decay rate.

(a) $A = 100(1.07)^t$ (b) $A = 5.3(1.054)^t$

(c) $A = 3500(0.93)^t$ (d) $A = 12(0.88)^t$

Solution (a): The initial amount is 100, and the function represents growth at a 7% rate. ■

Solution (b): The initial amount is 5.3, and the function represents growth at a 5.4% rate. ■

Solution (c): The initial amount is 3500, and the function represents decay at a 7% rate. ■

Solution (d): The initial amount is 12, and the function represents decay at a 12% rate. ■

1.5.10. The Hershey Company is the largest US producer of chocolate. In 2008, annual net sales were 5.1 billion dollars, and were increasing at a continuous rate of 4.3% per year.

(a) Write a formula for annual net sale, S, as a function of time, t, in years since 2008.

(b) Estimate annual net sales in 2015.

(c) Use a graph or trial and error to estimate the year in which annual net sales are expected to pass 8 billion dollars.

Solution (a): The growth factor is 1.043, so the formula for sales is $S(t) = 5.1(1.043)^t$, in billions of dollars. ∎

Solution (b): In 2015, $t = 2015 - 2008 = 7$, so the estimated annual net sales in 2015 are $S(7) = 5.1(1.043)^7 = 6.848$ billion dollars. ∎

Solution (c): Plugging whole numbers into $S(t)$, we see that $S(11) = 8.104$ is the first time that $S(t)$ is greater than 8. Hence, we expect net sales to pass 8 billion dollars in $2008 + 11 = 2019$. ∎

1.5.18. Give a possible formula for the function whose graph is shown (see text).

Solution: The shown graph passes through the points $(0, 30)$ and $(25, 6)$, and has the general formula $P(t) = P_0 a^t$. When $t = 0$, $P(0) = 30$, so $30 = P_0 a^0 = P_0$. Hence, $P_0 = 30$. We now find a. Since $P(25) = 6$, $6 = 30a^{25}$, so $a^{25} = \frac{6}{30} = \frac{1}{5}$. Therefore, $a = \left(\frac{1}{5}\right)^{1/25}$, so

$$P(t) = 30\left(\left(\frac{1}{5}\right)^{1/25}\right)^t = 30\left(\frac{1}{5}\right)^{t/25}.$$

∎

1.5.22. Find a formula for the number of zebra mussels in a bay as a function of the number of years since 2003, given that there were 2700 at the start of 2003 and 3186 at the start of 2004.

(a) Assume that the number of zebra mussels is growing linearly. Give units for the slope of the line and interpret it in terms of zebra mussels.

(b) Assume that the number of zebra mussels is growing exponentially. What is the percent rate of growth of the zebra mussel population?

Solution (a): If the population growth is linear, then the slope is

$$m = \frac{\Delta P}{\Delta t} = \frac{3186 - 2700}{2004 - 2003} = \frac{486}{1} = 486,$$

in mussels per year. Hence, the zebra mussel population is growing at a constant rate of 486 mussels per year. ∎

Solution (b): The growth factor from 2003 to 2004 is

$$a = \frac{3186}{2700} = 1.18.$$

Therefore, the percent growth rate is 18%. ∎

1.5.24. Below is a table of values from three functions, f, g, and h:

x	-2	-1	0	1	2
$f(x)$	12	17	20	21	18
$g(x)$	16	24	36	54	81
$h(x)$	37	34	31	28	25

(a) Which (if any) of the functions in the table could be linear? Find formulas for those functions.

(b) Which (if any) of the functions in the table could be exponential? Find formulas for those functions.

Solution (a): We compute the average rates of change for these functions in the intervals between these x values:

x	-2 to -1	-1 to 0	0 to 1	1 to 2
$f(x)$	5	3	1	-3
$g(x)$	8	12	18	27
$h(x)$	-3	-3	-3	-3

Since h is the only function channging at a constant average rate (-3), it is linear. Since $h(0) = 31$, a formula for $h(x)$ is $h(x) = 31 - 3x$. ∎

Solution (b): We compute the relative change of each function over these same intervals, using the formula $\frac{f(x_2)-f(x_1)}{f(x_1)}$

x	-2 to -1	-1 to 0	0 to 1	1 to 2
$f(x)$	$\frac{5}{12}$	$\frac{3}{17}$	$\frac{1}{20}$	$-\frac{1}{7}$
$g(x)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$h(x)$	$-\frac{3}{37}$	$-\frac{3}{34}$	$-\frac{3}{31}$	$-\frac{3}{28}$

Since $g(x)$ is the only function that shows constant relative change over intervals of the same length, it is the only function that could be exponential. Since its value at $x = 0$ is 36 and its growth factor is $a = 1 + \frac{1}{2} = \frac{3}{2}$, a formula for $g(x)$ is $g(x) = 36\left(\frac{3}{2}\right)^x$. ∎

1.5.28.

(a) Niki invested \$10,000 in the stock market. The investment was a loser, declining in value 10% per year each year for 10 years. How much was the investment worth after 10 years?

(b) After 10 years, the stock began to gain value at 10% per year. After how long will the investment regain its initial value (\$10,000)?

Solution (a): Losing 10% per year, the value of Niki's investment after t years is $10{,}000(0.9)^t$. When $t = 10$, Niki's investment will be worth

$$10{,}000(0.90)^{10} \approx 3486.78. \qquad \blacksquare$$

Solution (b): If t is the number of years of growth at 10%, then the value of the investment is $V(t) = 10{,}000(0.9)^{10}(1.1)^t$.

By trial and error, we see that $V(11) = 10{,}000(0.9)^{10}(1.1)^{11} \approx 9948.20$ and $V(12) = 10{,}000(0.9)^{10}(1.1)^{12} \approx 10{,}943.02$, so the investment takes an additional 12 years to regain its initial value (and more). $\blacksquare$