# Homework #1 Solutions

## Problems

- Section 1.1: 8, 10, 12, 14, 16
- Section 1.2: 2, 8, 10, 12, 16, 24, 26
- Extra Problems #1 and #2

1.1.8. Find $f(5)$ if $f(x) = 10x - x^2$.

*Solution:* Setting $x = 5$, $f(5) = 10(5) - 5^2 = 50 - 25 = 25$. ∎

1.1.10. Find $f(5)$ on the given graph. (See text.)

*Solution:* We check the height of the point on the graph above $x = 5$. Since this height is $y = 2$, $f(5) = 2$. ∎

1.1.12. Let $y = f(x) = x^2 + 2$.
(a) Find the value of $y$ when $x$ is zero.
(b) What is $f(3)$?
(c) What values of $x$ give $y$ a value of 11?
(d) Are there any values of $x$ that give $y$ a value of 1?

*Solution (a):* Setting $x = 0$, $y = f(0) = 0^2 + 2 = 0 + 2 = 2$. ∎

*Solution (b):* Setting $x = 3$, $y = f(3) = 3^2 + 2 = 9 + 2 = 11$. ∎

*Solution (c):* We set up the equation $x^2 + 2 = 11$. Then $x^2 = 9$, so $x = \pm\sqrt{9} = \pm 3$ (remember that square roots can be positive or negative). Hence, either $x = 3$ or $x = -3$. ∎

*Solution (d):* We set up the equation $x^2 + 2 = 1$. Then $x^2 = -1$. Since $x^2$ is a square, it is always greater than or equal to 0, so this equation has no solutions. ∎

1.1.14. The figure shows the amount of nicotine, $N = f(t)$, in mg, in a person's bloodstream as a function of the time, $t$, in hours, since the person finished smoking a cigarette.

(a) Estimate $f(3)$ and interpret it in terms of nicotine.

(b) About how many hours have passed before the nicotine level is down to 0.1 mg?

(c) What is the vertical intercept? What does it represent in terms of nicotine?

(d) If this function had a horizontal intercept, what would it represent?

*Solution (a):* From the graph, the value of $N$ at $t = 3$ is approximately 0.14 mg. ■

*Solution (b):* We look for where the graph first crosses the horizontal line at height $y = 0.1$. This appears to happen at $t = 4$. ■

*Solution (c):* The vertical intercept is the value of the function at $t = 0$. From the graph, this value is about 0.4 mg. This represents the amount of nicotine in the bloodstream immediately after the person finishes smoking the cigarette. ■

*Solution (d):* A horizontal intercept occurs when the height of the function is 0, so that would represent a complete absence of nicotine in the bloodstream. ■

1.1.16. A deposit is made into an interest-bearing account. Figure 1.9 shows the balance, $B$, in the account $t$ years later.

(a) What was the original deposit?

(b) Estimate $f(10)$ and interpret it.

(c) When does the balance reach $5000?

*Solution (a):* The original deposit is the value of the function at $t = 0$. From the graph, this value is $1000. ■

*Solution (b):* From the graph, $f(10)$ is approximately $2200. This is the amount of money in the account after 10 years. ■

*Solution (c):* From the graph, the balance reaches $5000 at about $t = 21$. ■

1.2.2. Determine the slope and the $y$-intercept of the line $3x + 2y = 8$.

*Solution:* We isolate $y$, to put the equation into $y = mx + b$ form:

$$\begin{aligned} 3x + 2y &= 8 \\ 2y &= -3x + 8 \\ \frac{2y}{2} &= \frac{-3x+8}{2} = -\frac{3}{2}x + \frac{8}{2} \\ y &= -\frac{3}{2}x + 4. \end{aligned}$$

Hence, the slope $m$ is $-\frac{3}{2}$, and the $y$-intercept $b$ is 4. ■

1.2.8. Find an equation of the line passing through $(4, 5)$ and $(2, -1)$.

*Solution:* We first find the slope of the line through the two points $(x_1, y_1) = (2, -1)$ and $(x_2, y_2) = (4, 5)$:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-1)}{4 - 2} = \frac{6}{2} = 3.$$

Using the point-slope formula with this slope and the point $(2, -1)$,

$$y - (-1) = 3(x - 2),$$

or $y + 1 = 3(x - 2)$, is an equation for the line.
Alternately, using the point $(4, 5)$ yields the equation

$$y - 5 = 3(x - 4).$$

Since both equations simplify to the equation $y = 3x - 7$, they are all the equivalent, and all provide valid equations for this line. ∎

1.2.10. Figure 1.22 shows lines $l_1$ through $l_5$.

(a) Which two lines have the same slope? Of these two lines, which has the larger $y$-intercept?

(b) Which two lines have the same $y$-intercept? Of these two lines, which has the larger slope?

*Solution (a):* Lines $l_2$ and $l_3$ have the same slope, since they are parallel. Line $l_2$ has the larger $y$-intercept. ∎

*Solution (b):* Lines $l_1$ and $l_3$ have the same $y$-intercept. Line $l_1$ has the larger slope. ∎

1.2.12. A cell phone company charges a monthly fee of \$25 plus \$0.05 per minute. Find a formula for the monthly charge, $C$, in dollars, as a function of the number of minutes, $m$, the phone is used during the month.

*Solution:* The monthly charge is $C(m) = 25 + 0.05m$. ∎

1.2.16. Let $y$ be the percent increase in annual US national production during a year when the unemployment rate changes by $u$ percent. (For example, $u = 2$ if unemployment increases from 4% to 6%.) Okun's law states that

$$y = 3.5 - 2u.$$

(a) What is the meaning of the number 3.5 in Okun's law?

(b) What is the effect on national production of a year when unemployment rises from 5% to 8%?

(c) What change in the unemployment rate corresponds to a year when production is the same as the year before?

(d) What is the meaning of the coefficient $-2$ in Okun's law?

*Solution (a):* 3.5 is the percent increase when $u = 0$, so 3.5% is the rate of growth of US national production assuming no change in the unemployment rate. ■

*Solution (b):* If unemployment rises from 5% to 8%, then $u = 8 - 5 = 3$. Hence, $y = 3.5 - 2(3) = 3.5 - 6 = -2.5$, so US national production should decrease by 2.5%. ■

*Solution (c):* If production is the same as in the previous year, $y = 0$, so $0 = 3.5 - 2u$. Hence, $2u = 3.5$, so $u = \frac{1}{2}(3.5) = 1.75$. Thus, unemployment that year increased by 1.75%. ■

*Solution (d):* The $-2$ indicates how the production rate changes with the unemployment. It is negative because as unemployment increases, the change in the US national production should decrease. ■

1.2.24. Annual sales of music compact discs (CDs) have declined since 2000. Sales were 942.5 million in 2000 and 384.7 million in 2008.

(a) Find a formula for annual sales, $S$, in millions of music CDs, as a linear function of the number of years, $t$, since 2000.

(b) Give units for and interpret the slope and the vertical intercept of the function.

(c) Use the formula to predict music sales in 2012.

*Solution (a):* We have two data points for the function $S(t)$, $(t_1, S_1) = (0, 942.5)$ and $(t_2, S_2) = (8, 384.7)$. The slope of the linear function is therefore

$$m = \frac{S_2 - S_2}{t_2 - t_1} = \frac{384.7 - 942.5}{8 - 0} = \frac{-557.8}{8} \approx -69.7,$$

rounding to significant figures. The vertical intercept is given by 942.5, so the function is

$$S(t) = 942.5 - 69.7t.$$

$S(t) = 942.5 - 69.725t$ is also acceptable if significant figures are ignored. ■

*Solution (b):* The units of the slope are in millions of CDs per year, and is negative because sales decrease over time. The vertical intercept is in millions of CDs and represents the number of CDs sold in the year 2000. ■

*Solution (c):* $S(2012) = 942.5 - 69.7(12) = 106.1$ million CDs is our sales estimate. (105.8 is also acceptable if 69.725 is used as the slope.) ■

1.2.26. The number of species of coastal dune plants in Australia decreases as the latitude, in °S, increases. There are 34 species at 11°S and 26 species at 44°S.

- Find a formula for the number, $N$, of species of coastal dune plants in Australia as a linear function of the latitude, $l$, in °S.
- Give units for and interpret the slope and the vertical intercept of the function.
- Graph this function between $l = 11°$S and $l = 44°$S. (Australia lies entirely between these latitudes.)

*Solution (a):* The slope of the linear function $N(l)$ is

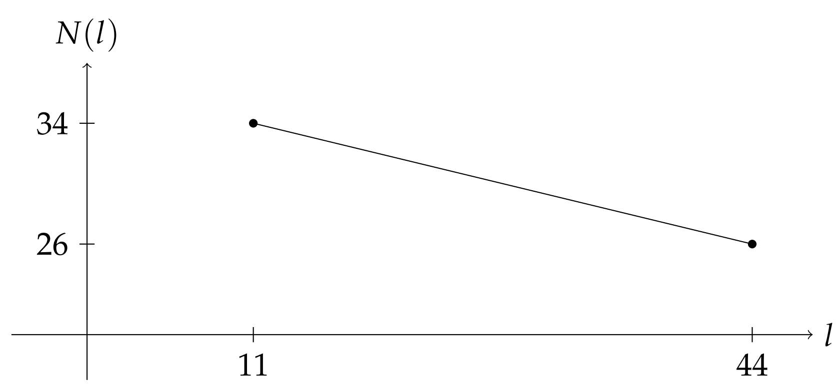
$$m = \frac{26 - 34}{44 - 11} = -\frac{8}{33}.$$

From the point-slope function formula,

$$N(l) = -\frac{8}{33}(l - 11) + 34 = -\frac{8}{33}l + \frac{8}{3} + 34 = -\frac{8}{33}l + \frac{110}{3}.$$ ■

*Solution (b):* The units of the slope are the number of species per degree of latitude, and are negative because the number of species decreases as the latitude increases. The vertical intercept has units the number of species, and gives the expected number of species at 0°S. ■

*Solution (c):* One such graph:

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EP 1.1. Find the value of $a$ so that the line $y = 1 + ax$ goes through the point $(2, 5)$.

*Solution:* Set $x = 2$ and $y = 5$. Then $5 = 1 + 2a$, so $2a = 4$, and $a = 2$. ■

EP 1.2. Find the value of $b$ so that the line $y = b - 2x$ goes through the point $(1, 3)$.

*Solution:* Set $x = 1$ and $y = 3$. Then $3 = b - 2(1) = b - 2$, so $b = 3 + 2 = 5$. ■