Midterm #2 Practice Problems

1. Compute the derivative of each function below. Simplify your answers where possible.

(a) $f(x) = x^3 + \dfrac{1}{x^3} + \sqrt[3]{x}$

(b) $h(t) = (4t^2 - t^3)e^t$

(c) $L(u) = \ln(u)\ln(\ln u)$

(d) $P(z) = \dfrac{e^{3z}}{z^{3/2}}$

(e) $Q(w) = e^{w^3 - 2w^2}$

2. Let $f(x) = 3x^5 - 20x^3$.

(a) Find $f'(x)$ and $f''(x)$.

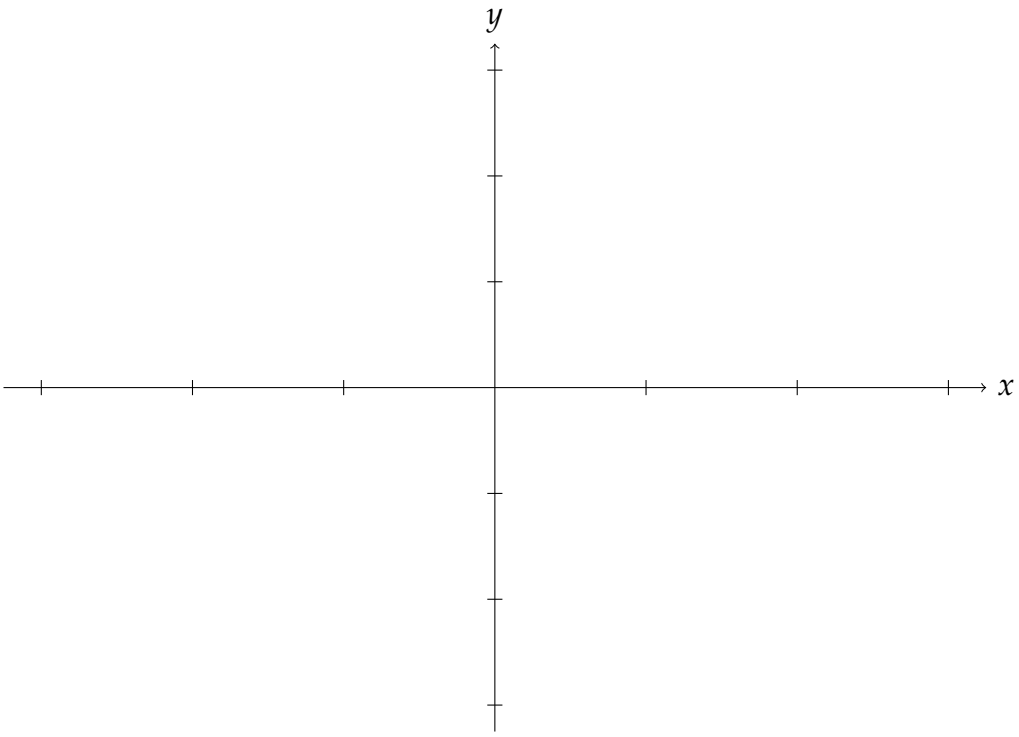
(b) Find the critical points of $f(x)$.

(c) Characterize each critical point as a local minimum, local maximum, or neither. Justify your answers.

(d) Find the intervals on which $f(x)$ is increasing and on which $f(x)$ is decreasing.

(e) Find the inflection points of $f(x)$. Justify your answers.

(f) Use the information in the parts above to make an accurate graph of $f(x)$ on the axes below. Indicate the scale on the x- and y-axes, and label the graph with the local extrema and inflection points.



3. Let $h(t) = (t^2 - 4)^{2/3}$.

(a) Find $h'(t)$. Simplify your answer.

(b) Find the critical points of $h(t)$.

(c) Find the intervals on which $h(t)$ is increasing and on which $h(t)$ is decreasing.

(d) Characterize each critical point as a local minimum, local maximum, or neither. Justify your answers.

4. Below are the values of $g(t)$ for certain values of t.

t	1	3	5	7	9	11	13
$g(t)$	0.7	1.9	2.7	3.1	2.9	1.5	-0.3

(a) Estimate $g'(3)$ and $g'(11)$. Explain your estimates.

(b) Do you expect $g''(t)$ to be positive or negative on this interval? Explain.

5. Our favorite budget steel mill, Bethlehem Steel, has made some changes to its steel prices. The cost in dollars of x tons of steel is now given by the function

$$C(x) = 2000 + 800x - 6x^2 + 0.05x^3.$$

(a) Find $C'(x)$.

(b) Evaluate $C(100)$ and $C'(100)$. Interpret your results, and include units.

(c) Find an equation of the tangent line to $C(x)$ at $x = 100$.

(d) Estimate $C(102)$.

6. Below are values of three functions, $r(x)$, $s(x)$, and $t(x)$, and their derivatives at different values of x.

x	$r(x)$	$s(x)$	$t(x)$	$r'(x)$	$s'(x)$	$t'(x)$
3	4	0	1	2	4	3
4	2	3	3	-2	6	2
5	3	4	4	-4	7	0

(a) Let $H(x) = r(s(x))$. Find $H'(4)$.

(b) Let $L(x) = \ln(t(x))$. Find $L'(3)$.

(c) Let $P(x) = r(x)s(x)$. Find $P'(5)$.

(d) Let $Q(x) = \dfrac{r(x)}{t(x)}$. Find $Q'(3)$.

(e) Let $V(x) = s(x)e^{r(x)}$. Find $V'(4)$.

7. On a hot summer's day, we launch a water balloon into the air from the roof of a building. The vertical position of the balloon is given by $y(t) = 35 + 30t - 5t^2$, in meters, where t is the time in seconds since the balloon was launched.

(a) What are the balloon's vertical velocity $v(t)$ and acceleration $a(t)$? Include units.

(b) How high up does the balloon go? At what time does the balloon reach its peak?

(c) How long does it take the balloon to hit the ground?

(d) What is the vertical velocity of the balloon when it hits the ground?

8. When a 200-milligram dose of the drug pretendozole ingested, the function

$$C(t) = \frac{60t}{t^3 + 16}$$

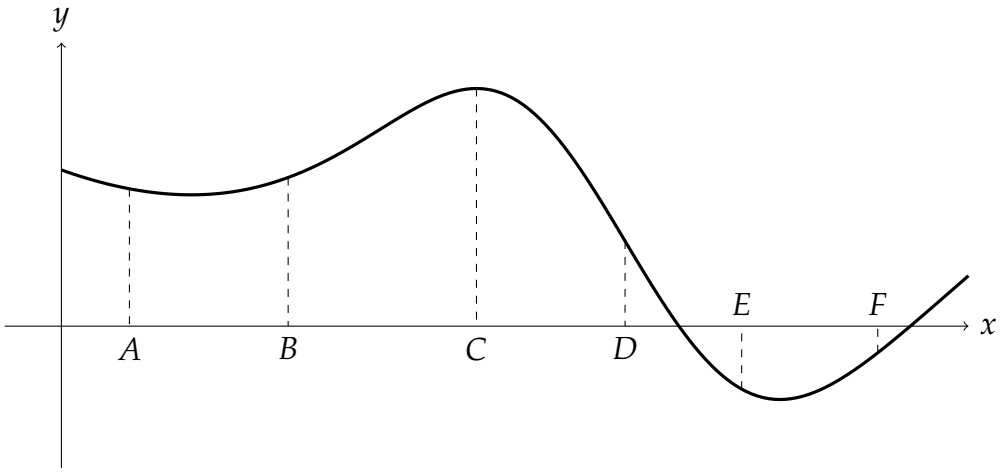
describes its concentration in the bloodstream t hours later, in mg/l.

(a) Find $C'(t)$. What are the units of this quantity?

(b) Evaluate $C'(1)$ and $C'(3)$. What do these values tell you about how $C(t)$ is changing?

(c) Find the time t when the maximum concentration occurs. What is the concentration at that maximum?

9. Below is the graph of a function $f(x)$, labeled with points A through F.



At which of the labeled points is

(a) $f(x)$ greatest?

(b) $f'(x)$ greatest?

(c) $f(x)$ smallest?

(d) $f'(x)$ smallest?

(e) $f'(x) = 0$?

(f) $f''(x) = 0$?