Midterm #2 — November 8, 2011, 8:30 to 10:00 PM

Name: _____

Circle your recitation:

R02 (Chira, Mon) R03 (Chira, Wed) R04 (Marcelo, Tue)

- **You have a maximum of $1\frac{1}{2}$ hours.** This is a closed-book, closed-notes exam. No calculators or other electronic aids are allowed.
- Read each question carefully. Show your work and justify your answers for full credit. You do not need to simplify your answers unless instructed to do so. Circle, box, or otherwise point out your final answer if it is not obvious.
- If you need extra room, use the back sides of each page. If you must use extra paper, make sure to write your name on it and attach it to this exam. Do *not* unstaple or detach pages from this exam.

Grading

1	/30
2	/15
3	/15
4	/15
5	/15
6	/10
Total	/100

1. (*30 points*) Find the derivative of each function below. Simplify your answers.

(a) (*5 points*) $f(x) = x^6 - 4x + 8$

(b) (*5 points*) $H(x) = (x^3 + 4x)^5$

(c) (*5 points*) $J(z) = \dfrac{z}{3 - z^2}$

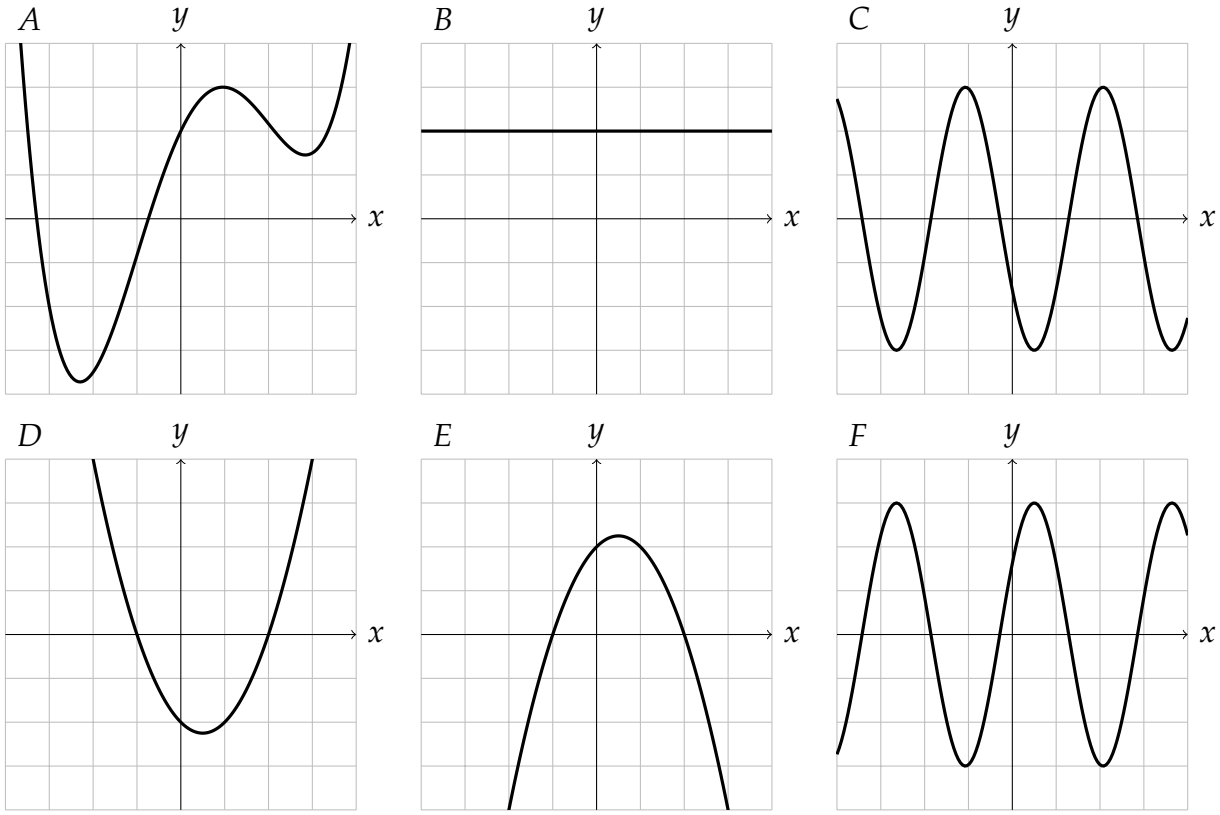
1., continued.

(d) (*5 points*) $h(v) = (v^2 - 2v + 2)e^v$

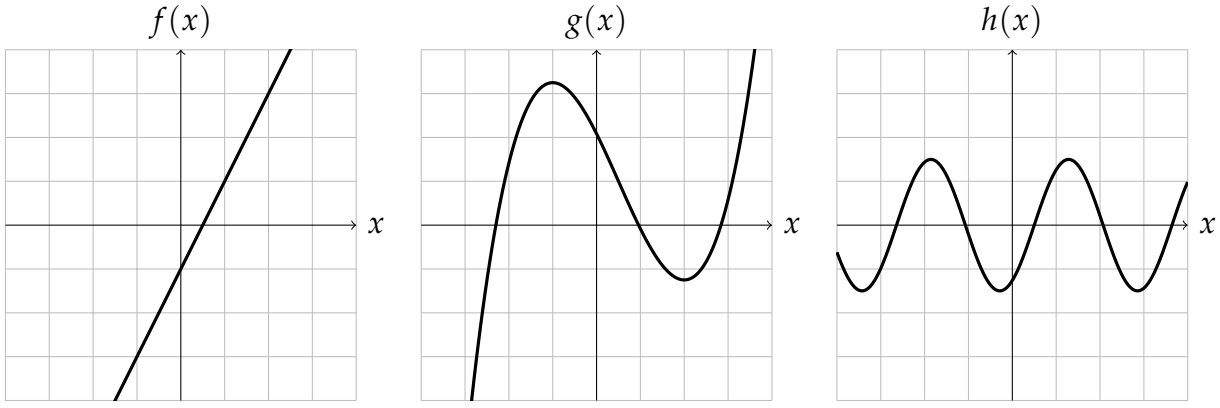
(e) (*5 points*) $A(u) = 8\sqrt[4]{u} + \dfrac{1}{u^3} - 5\ln u$

(f) (*5 points*) $V(t) = \dfrac{te^{t^2-2}}{t+3}$

2. (*15 points*) Below are the graphs of six functions, labeled A through F:



We also have graphs of three functions, $f(x)$, $g(x)$, and $h(x)$. Identify which of the graphs A–F are the graphs of their derivatives, $f'(x)$, $g'(x)$, and $h'(x)$.

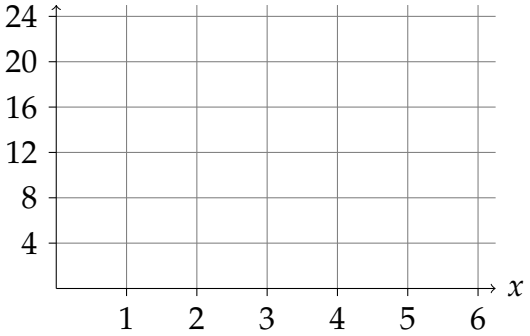


$f'(x)$: _____ $g'(x)$: _____ $h'(x)$: _____

3. (*15 points*) The function $P(x)$ gives the population of North Haverbrook, in thousands of people, where x is in years since 2005. The tangent line to the function $P(x)$ at $x = 3$ is given by $y = 20 - 2x$.

(a) (*4 points*) Find the values of $P(3)$ and $P'(3)$. What do these numbers mean? Include units in your explanation.

(b) (*4 points*) Suppose that $P''(3)$ is positive. On the axes to the right, sketch graphs of both $P(x)$ and the tangent line that are consistent with the above information.



(c) (*4 points*) Estimate $P(3.5)$.

(d) (*3 points*) Is your estimate in part (c) an underestimate or an overestimate? Explain.

4. (*15 points*) Let $f(x) = \dfrac{x^3}{3} - x^2 - 8x + 4$.

(a) (*3 points*) Find $f'(x)$. Simplify your answer.

(b) (*2 points*) Find $f''(x)$. Simplify your answer.

(c) (*3 points*) Find the critical points of $f(x)$.

4., continued.

(d) (*3 points*) For each critical point of $f(x)$, decide whether it is a local maximum, a local minimum, or neither. Explain your answers.

(e) (*2 points*) On which x-intervals is $f(x)$ increasing? Decreasing?

(f) (*2 points*) Find the inflection points of $f(x)$. Justify your answers.

5. (*15 points*) Below are the values of a function $w(t)$ at regularly spaced points between $t = 1$ and $t = 2$:

t	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$w(t)$	1.72	2.12	2.48	2.76	2.96	3.1	3.18	3.2	3.16	3.06	2.8

(a) (*8 points*) Estimate $w'(1.3)$ and $w'(1.9)$. Explain your estimates.

(b) (*4 points*) Do you think $w''(t)$ is positive, negative, or neither between $t = 1$ and $t = 2$? Why?

(c) (*3 points*) Estimate $w''(1.5)$. Explain how you computed your estimate.

6. (*10 points*) We sell tickets to a soccer game. We determine that if we set the price to be p dollars, the number of tickets we will sell is $q(p) = 200e^{(1-p/40)}$.

(a) (*3 points*) Write a formula for the total revenue $R(p)$ we collect, in terms of the price p.

(b) (*4 points*) Find the price p giving the maximum revenue and the revenue at this price.

(c) (*3 points*) Find the price p at which the revenue is decreasing the fastest.