Final Practice Problems

1. Find derivatives of the following functions:

(a) $f(x) = 2x^3 + 4x^2 - 3x + 5$

(b) $g(t) = \dfrac{t^4}{4} + \dfrac{1}{t^3} - \dfrac{2}{t^6}$

(c) $r(u) = e^{u^2 \ln u}$

(d) $Q(z) = \dfrac{e^z}{2 + \sqrt{z}}$

(e) $m(y) = 2^{3y - y^2}$

2. Compute the values of the following definite integrals:

(a) $\displaystyle\int_{-1}^{4} 2x + 1\, dx$

(b) $\displaystyle\int_{1}^{3} x^3 - 6x^2 + 12x - 8\, dx$

(c) $\displaystyle\int_{0}^{10} e^{0.2t}\, dt$

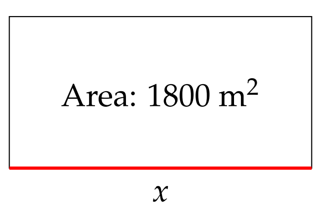
3. Find the general antiderivatives of the following functions:

(a) $f(x) = 6x^2 - 4x + 3$

(b) $g(t) = e^{t/6} - \dfrac{1}{t^3}$

(c) $h(z) = \dfrac{3}{z} + \dfrac{1}{\sqrt[3]{z}}$

4. We are building a rectangular garden of area 1800 m^2. On three of the sides, we will use fence that costs \$20 per meter, and on the last side we will use fence that costs \$60 per meter. Let x be the length of the side of the garden with the more expensive fence.



(a) Find an expression for the cost C of the fence in terms of x.

(b) Find the value of x that minimizes the cost of the fence.

(c) At the minimum cost, what are the dimensions of the garden, and how much does the entire fence cost?

5. Let $f(x) = \frac{1}{4}x^4 - x^3 - 2x^2 + 3$.

(a) Find $f'(x)$ and $f''(x)$.

(b) Find the critical points of $f(x)$.

(c) Characterize each critical point as a local minimum, local maximum, or neither. Justify your answers.

(d) Find the intervals on which $f(x)$ is increasing and on which $f(x)$ is decreasing.

(e) Find the *global* maximum and minimum values of $f(x)$ on the interval $[-2, 4]$, and the x-values where they occur.

(f) Find the inflection points of $f(x)$. Justify your answers.

6. Find the area of the region enclosed by the curves $y = x^2$ and $y = 8\sqrt{x}$.

7. Let $f(x) = x^3 - \frac{1}{x^2}$.

(a) Find the general antiderivative $G(x)$ of $f(x)$.

(b) Find the antiderivative $G(x)$ of $f(x)$ satisfying $G(2) = \frac{5}{2}$.

8. Let $g(x) = x^{4/3}$.

(a) Find $g'(x)$.

(b) Find the equation of the tangent line to the graph $y = x^{4/3}$ at $x = 8$.

(c) Use the tangent line to approximate $(8.03)^{4/3}$.

9. A water pipe bursts. The flow through the burst pipe wall is given, in liters per second, by $F(t)$, where t is the time in seconds since the pipe burst. A table of these flow rates is given below:

t (s)	0	1	2	3	4
$F(t)$ (l/s)	40	30	24	18	14

Using the average of a left-hand and a right-hand Riemann sum, estimate the volume of water that escapes the pipe during the first 4 seconds.

10. We sell bushels of apples from our farm in Riverhead, NY. We discover that if we sell a bushel at $20, we can sell 2000 bushels, but if we decrease the price to $18, we sell 2400.

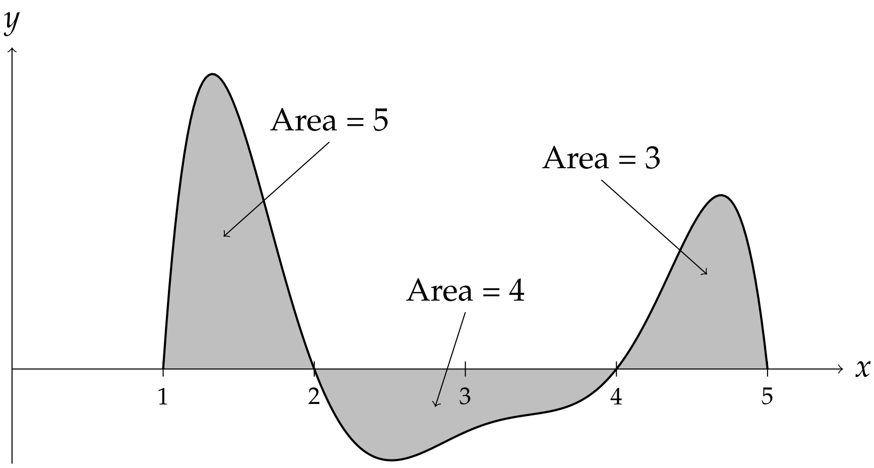
(a) Find the price $p(q)$ as a function of the quantity sold, q.

(b) Find the revenue $R(q)$ as a function of q.

(c) Find the quantity q that maximizes the revenue. What is the price we should charge to sell that quantity?

(d) We calculate our costs to be $C(q) = 5000 + 10q$. Find the quantity that maximizes the profit that we make. What is the price we should charge, and what is our profit?

11. Below is the graph of a function $g(x)$ on the interval $[1, 5]$.



(a) Find $\int_1^4 g(x)\,dx$.

(b) Find $\int_2^5 g(x)\,dx$.

(c) Find $\int_1^5 g(x)\,dx$.

(d) Find the total area of the shaded region enclosed by the curve and the x-axis.

(e) Find the average value of $g(x)$ from $x = 1$ to $x = 5$.

12. Let $z(t) = \dfrac{36}{(2z+1)^2}$.

(a) Find the left-hand Riemann sum of $z(t)$ from 0 to $\dfrac{3}{2}$ with $n = 3$.

(b) Find the right-hand Riemann sum of $z(t)$ from 0 to $\dfrac{3}{2}$ with $n = 3$.

(c) Use these Riemann sums to estimate $\int_0^{3/2} z(t)\,dt$.

(d) An antiderivative of $z(t)$ is $w(t) = -\dfrac{18}{2z+1}$. Compute the exact value of $\int_0^{3/2} z(t)\,dt$. What is the error in your estimate from part (c)?